Unit 15 · Measure of Dispersion

ANSWER KEY · FOR TEACHERS

Full marks 35

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 3

1. variance
2. coefficient of variation
3. consistent
4. skewness
5. positively
6. median

Each carries 0.5 mark.

2 True or false

U · 3

i. ✓ **True**

ii. ✗ **False**: It multiplies σ by $|-2| = 2$ (and the variance by 4); a standard deviation is never negative.

iii. ✗ **False**: Compare the coefficients of variation: σ alone ignores the sizes of the values.

iv. ✓ **True**

v. ✗ **False**: $\overline{X} - M_o = -4 < 0$: it is negatively skewed.

vi. ✓ **True**

Each carries 0.5 mark.

3 Multiple choice

i. K · 1

a. $\frac{\sigma}{\overline{X}} \times 100\%$

The standard deviation as a percentage of the mean. The third is Karl Pearson's coefficient of skewness.

ii. U · 1

a. 4

Halving halves σ; subtracting 5 moves every value alike and changes nothing: $\sigma = 4$. -1 and 3 wrongly subtract the 5 from σ, and 16 doubles instead of halving.

iii. U · 1

a. more than the median income

A tail of a few large incomes: positively skewed, and the large values pull the mean above the median (usually $M_o < M_d < \overline{X}$).

iv. U · 1

b. all the values are equal

σ^2 is a mean of squares, which are never negative; it is 0 only when every distance from the mean is 0, that is when all the values are equal (to the mean, which need not be 0: 5, 5, 5 has $\sigma = 0$).

v. A · 1

a. 25

$\sigma = 20\% \times 25 = 5$, so the variance is $5^2 = 25$. 5 is σ itself; 125 is 5×25; 0.8 is $\frac{20}{25}$.

vi. HA · 1

d. the coefficient of variation

Multiplying every value by 1.1 multiplies the mean and σ by 1.1 (and the variance by 1.21), so their ratio, the C.V., is unchanged. (Adding 10 marks to each would do the opposite: σ the same, the C.V. smaller.)

4 Short answer

U · 2

MARKING SCHEME

✓ No: the C.V.s (103.8% and 104.0%) are practically equal.

✓ Pokhara's larger σ comes from its larger mean, not from a more uneven year.

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5 Short questions

a. **ANSWER** A · 5

Pemba: $\overline{X} = \frac{300}{6} = 50$; distances $-2, 2, 0, 4, -4, 0$, squares totaling 40; $\sigma = \sqrt{\frac{40}{6}} \approx 2.58$;

C.V. $= \frac{2.58}{50} \times 100\% \approx 5.2\%$.

Rubina: $\overline{X} = \frac{240}{6} = 40$; distances $-10, 12, -16, 20, -4, -2$, squares totaling 920; $\sigma = \sqrt{\frac{920}{6}} \approx 12.38$;

C.V. $= \frac{12.38}{40} \times 100\% \approx 31.0\%$.

Pemba's sales are far more consistent (5.2% against 31.0%).

MARKING SCHEME

- ✓ Pemba: mean 50, $\sigma \approx 2.58$.
- ✓ Pemba's C.V. about 5.2%.
- ✓ Rubina: mean 40, $\sigma \approx 12.38$.
- ✓ Rubina's C.V. about 31.0%.
- ✓ Pemba's sales are more consistent (smaller C.V.).

b. **ANSWER** A · 5

Wrong totals: $\Sigma x = 20 \times 50 = 1000$; $\Sigma x^2 = n(\sigma^2 + \overline{X}^2) = 20(16 + 2500) = 50320$.
Correct totals: $\Sigma x = 1000 - 64 + 46 = 982$; $\Sigma x^2 = 50320 - 4096 + 2116 = 48340$.

Correct mean $= \frac{982}{20} = 49.1$; correct $\sigma = \sqrt{\frac{48340}{20} - 49.1^2} = \sqrt{2417 - 2410.81} = \sqrt{6.19} \approx 2.49$.

MARKING SCHEME

- ✓ Wrong $\Sigma x = 1000$ and $\Sigma x^2 = 20(16 + 2500) = 50320$.
- ✓ Correct $\Sigma x = 982$.
- ✓ Correct $\Sigma x^2 = 48340$.
- ✓ Correct mean 49.1.
- ✓ Correct $\sigma = \sqrt{6.19} \approx 2.49$.

6 Long question

ANSWER A · 8

a. With $A = 45$, $h = 10$: $d' = -3, -2, -1, 0, 1, 2$; fd': $-6, -12, -8, 0, 20, 24$, so $\Sigma fd' = 18$; fd'^2: 18, 24, 8, 0, 20, 48, so $\Sigma fd'^2 = 118$; $N = 60$.

$\overline{X} = 45 + \frac{18}{60} \times 10 = 48$ marks. $\sigma = \sqrt{\frac{118}{60} - \left(\frac{18}{60}\right)^2} \times 10 = \sqrt{1.9667 - 0.09} \times 10 \approx 13.70$ marks.

b. C.V. $= \frac{13.70}{48} \times 100\% \approx 28.5\%$.

c. Cumulative frequencies 2, 8, 16, 28, 48, 60; $\frac{N}{2} = 30$ falls in 50–60: $M_d = 50 + \frac{30 - 28}{20} \times 10 = 51$.

Modal class 50–60 ($f_1 = 20, f_0 = 12, f_2 = 12$): $M_o = 50 + \frac{20 - 12}{40 - 12 - 12} \times 10 = 55$.

d. From the mode: $S_k = \frac{48 - 55}{13.70} \approx$ -0.51. From the median: $S_k = \frac{3(48 - 51)}{13.70} \approx$ -0.66. Both negative: a few low scores pull the mean below the median and the mode, as in an easy test.

MARKING SCHEME

- ✓ a. The step deviation columns, $\Sigma fd' = 18$ and $\Sigma fd'^2 = 118$.
- ✓ Mean 48.
- ✓ $\sigma \approx 13.70$.
- ✓ b. C.V. about 28.5%.
- ✓ c. Median 51.
- ✓ Mode 55.
- ✓ d. $S_k \approx -0.51$ (mode) and -0.66 (median).
- ✓ Negatively skewed: a few low marks pull the mean below the median and the mode.

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7 Think

ANSWER HA · 3

No. In °F Jumla's mean is $1.8 \times 0.5 + 32 = 32.9$ °F and its $\sigma = 1.8 \times 3 = 5.4$ °F, so its C.V. is about 16.4%, not 600%: the same nights give a quite different C.V. on another scale. The C.V. means something only for data measured from a true zero (rain, weight, money); 0 °C is not "no temperature", and a mean near 0 makes the C.V. huge for no real reason. Both places have $\sigma = 3$ °C in the same unit, so their nights are equally steady.

MARKING SCHEME

- ✓ In °F: mean 32.9, $\sigma = 5.4$, C.V. about 16.4%: it changes with the scale.
- ✓ The C.V. needs a true zero; 0 °C is not one, and a mean near 0 inflates it.
- ✓ Compare the σs (same unit): equally steady.

8 Activity: How long is a word?

(not marked)

WHAT STUDENTS SHOULD FIND

Most words are short (2 to 5 letters) and a few are long, so the data are positively skewed: the mean is a little above the median and the mode, and S_k is positive. A paragraph of science words gives a larger mean and σ, but usually still a tail to the right.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	4	8	19	4	**35**

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