Unit 14 · Vectors

WORKSHEET

Full marks 35

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 35

1 Key words [3]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

basis	**collinear**	**coplanar**	**dependent**	**independent**	**linear combination**	**resultant**
unit vector						

Vectors parallel to one line are _____; vectors parallel to one plane are _____. A vector $x\vec{a} + y\vec{b}$, where x and y are numbers, is a _____ of $\vec{a}$ and $\vec{b}$. If $x_1\vec{a_1} + x_2\vec{a_2} + x_3\vec{a_3} = 0$ for some numbers not all zero, the vectors are linearly _____; if it happens only when every number is zero, they are linearly _____. Three non-coplanar vectors act as axes for space: they form a _____.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. The zero vector is collinear with every vector. ☐

ii. Any three vectors are coplanar. ☐

iii. $3\vec{i} - \vec{j} + 2\vec{k}$ and $-6\vec{i} + 2\vec{j} - 4\vec{k}$ are collinear. ☐

iv. Since $0\vec{a} + 0\vec{b} + 0\vec{c} = \vec{0}$, the vectors $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent. ☐

v. Four vectors in space can be linearly independent. ☐

vi. If two of three vectors are collinear, the three are linearly dependent. ☐

3 Multiple choice [6]

Choose the correct option for each of the following questions.

i. Three vectors in space are linearly dependent exactly when they are: [1]

a. collinear **b.** coplanar

c. non-coplanar **d.** perpendicular to each other

ii. $3\vec{i} - 2\vec{j} + m\vec{k}$ and $-6\vec{i} + 4\vec{j} + 8\vec{k}$ are collinear when m is: [1]

a. 4 **b.** -4

c. -16 **d.** -2

iii. Which of these sets of vectors is linearly independent? [1]

a. $\vec{i} + 2\vec{j}, 2\vec{i} + 4\vec{j}$

b. $\vec{i}, \vec{j}, \vec{i} + \vec{j}$

c. $\vec{i}, \vec{j}, 0$

d. $\vec{i} + \vec{j}, \vec{j} + \vec{k}, \vec{i}$

iv. $\vec{a} = \vec{i} + 2\vec{j}$ and $\vec{b} = 3\vec{i} - \vec{j}$. Then $5\vec{i} + 3\vec{j}$ is: [1]

a. $2\vec{a} + \vec{b}$ **b.** $\vec{a} + 2\vec{b}$

c. $2\vec{a} - \vec{b}$ **d.** $3\vec{a} - \vec{b}$

v. $\vec{i} + 2\vec{j} - \vec{k}$, $2\vec{i} + \vec{j} + \lambda\vec{k}$ and $3\vec{i} + 3\vec{j} + \vec{k}$ are linearly dependent when λ is: [1]

a. -2 **b.** 0

c. 1 **d.** 2

vi. $\vec{u}$ and $\vec{v}$ are non-collinear, and $(x-1)\vec{u} + (2-y)\vec{v}$ is collinear with $\vec{u} + \vec{v}$. Then: [1]

a. $x + y = 3$ **b.** $x = 1$ and $y = 2$

c. $x - y = 1$ **d.** x and y can be any numbers

4 Short answer [2]

Write a short answer to the following question.

Nirjala says that (0, 2, 1) and (0, 4, 2) can't be tested for collinearity, because $\frac{0}{0}$ has no meaning. Is she right? Explain.

5 Short questions [10]

Answer the following questions. Show your working.

a. Show that $A(1, 0, -2)$, $B(3, -1, 1)$ and $C(7, -3, 7)$ are collinear. [3] [5]

b. Find the ratio $AB : BC$ and a unit vector along $\overrightarrow{AB}$. [2]

Answer:

b. $\vec{a} = \vec{i} + 2\vec{j} - \vec{k}, \vec{b} = 2\vec{i} - \vec{j} + \vec{k}$ and $\vec{c} = \vec{i} + \vec{j} + 2\vec{k}$. **[5]**

a. Show that $\vec{a}, \vec{b}, \vec{c}$ are linearly independent. [2]

b. Express $\vec{r} = 4\vec{i} - \vec{j} - \vec{k}$ as a linear combination of $\vec{a}, \vec{b}, \vec{c}$. [3]



Answer: _____

6 Long question **[8]**

Answer the following question. Show your working.

$\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, and $\vec{u} = \vec{a} + 2\vec{b} - \vec{c}, \vec{v} = 2\vec{a} - \vec{b} + 3\vec{c}, \vec{w} = 4\vec{a} + 3\vec{b} + \lambda\vec{c}$.

a. Find λ so that $\vec{u}, \vec{v}, \vec{w}$ are linearly dependent. [3]

b. For that λ, express $\vec{w}$ as a combination of $\vec{u}$ and $\vec{v}$. [2]

c. Prove by vectors that the line joining the midpoints of two sides of a triangle is parallel to the third side and half as long. [3]



Answer: _____

7 Think [3]

Anjila says: “Any four vectors in space are dependent, so each of them is a combination of the other three.” Is she right? Explain with an example.

8 Activity: Balancing a flame (not marked)

A gas stove burns methane: $x\,CH_4 + y\,O_2 \rightarrow z\,CO_2 + w\,H_2O$. Write each molecule as its count of (carbon, hydrogen, oxygen) atoms, then find the smallest whole numbers that keep every atom.

Molecule	Carbon	Hydrogen	Oxygen
CH_4			
O_2			
CO_2			
H_2O			

The equations for carbon, hydrogen and oxygen: _____. The balance: $x =$ ___, $y =$ ___, $z =$ ___, $w =$ ___.