Unit 13 · Coordinates in Space

HARDER PRACTICE QUESTIONS

Full marks 30

Write your answers on separate paper. Show your working.

1 A point from its distances to the axes [6]

A point $P(x, y, z)$ is $\sqrt{13}$ units from the X-axis, $\sqrt{10}$ from the Y-axis and $\sqrt{5}$ from the Z-axis.

a. Show that the squares of a point's distances from the three axes add up to $2\,OP^2$. [2]

b. Find OP, and every point P that fits. How many are there? [2]

c. Could a point be 3 units from the X-axis, 4 from the Y-axis and 8 from the Z-axis? Give a reason. [2]

2 Equally far from three points [6]

$A(4, 1, 1)$, $B(1, 1, 2)$ and $C(0, -3, 1)$.

a. Find the point P in the XY-plane that is equally far from A, B and C, and the common distance. [4]

b. Now take $D(-2, 1, 3)$ instead of C. Try the same method for A, B and D, and explain what goes wrong and why. [2]

3 Where a point lies on a line [6]

A straight path passes through $A(1, -2, 3)$ and $B(3, 2, -1)$.

a. The point C on the line AB has $z = 7$. Find C. Is it between A and B? [2]

b. Find where the line crosses each of the three coordinate planes. [2]

c. Put A, B, C and the three crossing points in order along the line, and name the octants the line passes through, in order from C towards B and beyond. [2]

4 Can the angles be like that? [6]

A line makes angles α, α and β with the X-, Y- and Z-axes.

a. Show that $\sin^2\beta = 2\cos^2\alpha$. [2]

b. Saurav reads that such a line also has $\sin^2\beta = 3\sin^2\alpha$. He finds $\cos^2\alpha = \frac{3}{5}$. Check his algebra, then show that no line fits. [2]

c. For which values of k can a line have $\sin^2\beta = k\sin^2\alpha$? When $k = 1$, what are the line's possible direction ratios? [2]

5 A cable car's straight line [6]

Take the lower station of a cable car (made-up numbers) as the origin, the X-axis east, the Y-axis north and the Z-axis up, in meters. The upper station is at $U(1200, 900, 800)$, and the cable is taken as straight.

a. How long is the cable, and at what angle above the horizontal does it climb? [3]

b. A tower stands on a hillside 300 m east and 225 m north of the lower station, its foot 150 m higher than the station. How high above the tower's foot does the cable pass? [2]

c. A second straight cable from the same lower station runs due north and climbs at the same angle to a station also 800 m higher. Where is that station? [1]