Unit 13 · Coordinates in Space

ANSWER KEY · FOR TEACHERS

Full marks 35

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 3

1. origin
2. coordinate plane
3. direction cosines
4. direction ratios
5. right-handed
6. collinear

Each carries 0.5 mark.

2 True or false

U · 3

i. ✓ **True**

ii. ✗ **False**: Every point in the ZX-plane has $y = 0$; $z = 0$ is the XY-plane.

iii. ✗ **False**: $1 + 4 + 9 \neq 1$: they can only be direction ratios. The direction cosines are $\pm\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$.

iv. ✗ **False**: That needs $l^2 + m^2 + n^2 = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$, not 1: no line can.

v. ✓ **True**

vi. ✗ **False**: It is $\sqrt{y^2 + z^2}$; $|x|$ is its distance from the YZ-plane.

Each carries 0.5 mark.

3 Multiple choice

i. **b.** $(0, y, 0)$ — K · 1

On the Y-axis a point is in both the XY- and YZ-planes, so $z = 0$ and $x = 0$. $(x, 0, z)$ is the ZX-plane, $(x, y, 0)$ the XY-plane and $(0, y, z)$ the YZ-plane.

ii. **b.** 7 — U · 1

$\sqrt{4 + 36 + 9} = \sqrt{49} = 7$. -1 adds the coordinates with their signs, 11 adds their sizes, and 49 is the square.

iii. **b.** 5 — U · 1

The foot of the perpendicular on the Y-axis is (0, 7, 0), so the distance is $\sqrt{3^2 + (-4)^2} = 5$. 7 is the distance from the ZX-plane and $\sqrt{74}$ is OP.

iv. **a.** $\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}$ — U · 1

Divide by $\sqrt{16 + 16 + 4} = 6$. The second loses the minus sign; the third divides by 10, the sum of the sizes; the fourth divides by 36, forgetting the root. Only the first has squares adding to 1.

v. **d.** $90°$ — A · 1

$n^2 = 1 - \frac{1}{2} - \frac{1}{2} = 0$, so $n = 0$ and $\gamma = 90°$: the line lies level, in a plane parallel to the XY-plane.

vi. **a.** -1 — HA · 1

$\cos 2\alpha = 2\cos^2\alpha - 1 = 2l^2 - 1$, and so on: the sum is $2(l^2 + m^2 + n^2) - 3 = 2 - 3 = -1$, for every line.

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4 Coordinates in the box

MARKING SCHEME U · 2

✓ $L(2, 3, 0)$ and $M(0, 3, 4)$.

✓ $N(2, 0, 4)$ and $C(0, 0, 4)$.

5 Short questions

a. **ANSWER** A · 5

a. $AB = \sqrt{36 + 4 + 9} = \sqrt{49} = 7$.

b. Let $P(0, 0, z)$. $PA^2 = PB^2$: $4 + 1 + (z - 3)^2 = 16 + 1 + z^2$, so $z^2 - 6z + 14 = z^2 + 17$, $-6z = 3$, $z = -\frac{1}{2}$. The point is $\left(\mathbf{0, 0, -\frac{1}{2}}\right)$. Check: $PA^2 = 4 + 1 + 12.25 = 17.25 = 16 + 1 + 0.25 = PB^2$. ✓

MARKING SCHEME

✓ $AB^2 = 36 + 4 + 9$.

✓ $AB = 7$.

✓ The point taken as $(0, 0, z)$ and $PA^2 = PB^2$ written.

✓ $4 + 1 + (z - 3)^2 = 16 + 1 + z^2$ simplified to $-6z = 3$.

✓ $\left(0, 0, -\frac{1}{2}\right)$.

b. **ANSWER** A · 5

a. Direction ratios of AB: 2, 3, 1; of AC: 6, 9, 3. They are proportional ($\frac{6}{2} = \frac{9}{3} = \frac{3}{1} = 3$), so $AB \parallel AC$; and the two segments share the point A. So A, B, C **are collinear**.

b. The ratio is positive, so B and C are on the same side of A, with C three times as far: B **lies between** A **and** C. Check: $AB = \sqrt{14}$, $BC = \sqrt{16 + 36 + 4} = 2\sqrt{14}$, $AC = \sqrt{36 + 81 + 9} = 3\sqrt{14}$, and $AB + BC = AC$. ✓

MARKING SCHEME

✓ Direction ratios of AB: 2, 3, 1, and of AC: 6, 9, 3 (or of BC: 4, 6, 2).

✓ They are proportional.

✓ A common point named, so collinear.

✓ B lies between A and C.

✓ The distances $\sqrt{14}$, $2\sqrt{14}$, $3\sqrt{14}$ with $AB + BC = AC$.

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6 Long question

ANSWER A · 8

a. $AB^2 = 4 + 1 + 4 = 9$, $BC^2 = 4 + 4 + 1 = 9$, $AC^2 = 0 + 9 + 9 = 18$. $AB = BC = 3$ and $AB^2 + BC^2 = AC^2$: **right-angled at B, and isosceles**. ✓

b. AC: direction ratios 0, 3, −3, length $3\sqrt{2}$. Direction cosines **0, $\frac{1}{\sqrt{2}}$, $-\frac{1}{\sqrt{2}}$**: angles **90°, 45°, 135°**.
AC is square to the X-axis: it lies in a plane parallel to the YZ-plane, since A and C both have $x = 3$.

c. Area $= \frac{1}{2} \times AB \times BC = \frac{1}{2} \times 3 \times 3 = \frac{9}{2}$ square units.

d. B to C is the step (+2, +2, −1), so $D = A + (2, 2, -1) = (5, 3, 4)$. Direction ratios of AD: $5 - 3, 3 - 1, 4 - 5 = 2, 2, -1$, the same as BC's, $3 - 1, 4 - 2, 2 - 3 = 2, 2, -1$. ✓ Then $CD^2 = 4 + 1 + 4 = 9$ and $AD^2 = 9$: all four sides 3, the angle at B right, a square.

MARKING SCHEME

- ✓ $AB^2 = 9$, $BC^2 = 9$, $AC^2 = 18$.
- ✓ $AB = BC$ and $AB^2 + BC^2 = AC^2$: right-angled at B, isosceles.
- ✓ Direction ratios of AC: 0, 3, −3; direction cosines 0, $\frac{1}{\sqrt{2}}$, $-\frac{1}{\sqrt{2}}$.
- ✓ Angles 90°, 45°, 135°.
- ✓ The area is half the product of the two equal sides.
- ✓ Area $= \frac{9}{2}$ square units.
- ✓ $D = A + (C - B) = (5, 3, 4)$.
- ✓ AD and BC both have direction ratios 2, 2, −1.

7 Think

ANSWER HA · 3

No. Proportional direction ratios only mean that AB and CD are **parallel**. For one line the two segments must also share a point, or the line joining a point of one to a point of the other must have the same direction. AC has direction ratios 1, 0, 0, not proportional to 1, 2, 2, so C is not on line AB: the two lines are parallel and distinct.

MARKING SCHEME

- ✓ No: proportional direction ratios mean only parallel.
- ✓ She must also check a common point, or that AC (or AD) has the same direction ratios.
- ✓ AC: 1, 0, 0, not proportional, so parallel but distinct lines.

8 Activity: Your classroom in coordinates

(not marked)

WHAT STUDENTS SHOULD FIND

The computed and measured distances agree to within a few centimeters if the room's corner is square; the difference comes from the tape, the sag of the string and how square the corner is. The squares of the direction cosines add to 1 up to rounding, a check that the three coordinates were measured along three directions at right angles.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	4	8	19	4	**35**

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