Unit 12 · Pair of Straight Lines

WORKSHEET

Full marks 40

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 40

1 Key words [3]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

coincident	**half**	**homogeneous**	**imaginary**	**origin**	**parallel**	**perpendicular**	**tangent**

An equation all of whose terms have the same degree is _____; $ax^2 + 2hxy + by^2 = 0$ is a pair of lines through the _____. Its lines are real and different if $h^2 > ab$, _____ if $h^2 = ab$, and _____ if $h^2 < ab$, when only $(0, 0)$ is a real point of it. They are _____ when $a + b = 0$. In $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, h is _____ the coefficient of xy.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. $x^2 - y^2 = 0$ is a pair of perpendicular lines. ☐

ii. In $2x^2 + 3xy - 2y^2 - 5x + 5y - 3 = 0$, $h = 3$. ☐

iii. Every equation of the second degree in x and y represents a pair of lines. ☐

iv. The lines of $3x^2 + 7xy + 2y^2 + 5x + 5y + 2 = 0$ make the same angle as those of $3x^2 + 7xy + 2y^2 = 0$. ☐

v. The two lines of a homogeneous second-degree equation can be parallel and different. ☐

vi. The bisectors of the angles between the lines $2x^2 + 5xy + 2y^2 = 0$ are $xy = 0$. ☐

3 Multiple choice [6]

Choose the correct option for each of the following questions.

i. The lines of $ax^2 + 2hxy + by^2 = 0$ are perpendicular when: [1]

a. $h^2 = ab$ **b.** $a + b = 0$

c. $h = 0$ **d.** $a = b$

ii. $ax^2 + 2hxy + by^2 = 0$ represents two coincident lines when: [1]

a. $h^2 = ab$ **b.** $h^2 > ab$

c. $h^2 < ab$ **d.** $a + b = 0$

iii. The lines represented by $x^2 - 7xy + 12y^2 = 0$ are: [1]

a. $x = 3y$ and $x = 4y$ **b.** $y = 3x$ and $y = 4x$

c. $x = -3y$ and $x = -4y$ **d.** $x = 3y$ and $x = -4y$

iv. The acute angle between the lines of $x^2 + 4xy + y^2 = 0$ is: [1]

a. $30°$ **b.** $45°$

c. $60°$ **d.** $90°$

v. $2x^2 + 7xy + 3y^2 + 8x + 14y + k = 0$ represents a pair of lines when k is: [1]

a. 6 **b.** 8

c. -8 **d.** 12

vi. The slope of one line of $ax^2 + 2hxy + by^2 = 0$ is twice the other's. Then: [1]

a. $8h^2 = 9ab$ **b.** $9h^2 = 8ab$

c. $h^2 = 2ab$ **d.** $4h^2 = 9ab$

4 Short answers [11]

Write short answers to the following questions.

a. Find the single equation of the pair of lines $y = 3x$ and $2x + y = 0$. [2]

b. Lhakpa found the acute angle between the lines $x^2 - 6xy + 5y^2 = 0$ as $\tan\theta = \frac{2\sqrt{36-5}}{1+5} = \frac{\sqrt{31}}{3}$. [2]

Find his mistake and the right answer.

c. One of the lines of $2x^2 + kxy - 3y^2 = 0$ is $y = 2x$. Find k and the other line. [2]

d. Find k so that $12x^2 - 10xy + 2y^2 + 11x - 5y + k = 0$ represents a pair of lines. Are the lines real? [3]

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e. Explain why the equation $ax^2 + 2hxy + by^2 = 0$, when it has a point other than the origin, holds the whole line through that point and the origin. **[2]**

5 Structured question **[5]**

Answer all the parts.

a. Show that $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$ represents a pair of lines. [2]
b. Find the two lines and the acute angle between them. [3]

6 Long question **[8]**

Write a long answer to the following question.

a. Prove that the bisectors of the angles between the lines $ax^2 + 2hxy + by^2 = 0$ are $h(x^2 - y^2) = (a - b)xy$, and that they are perpendicular. [4]
b. Find the bisectors of the angles between the lines $7x^2 - 8xy + y^2 = 0$, and show that one of them makes equal angles with the two lines. [2]
c. Find the same bisectors from Unit 11's formula for the bisectors of two lines. [2]

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7 Think [4]

In the unit's drum model the ears are on the X-axis, and a sound from a direction at angle φ to the X-axis reaches the left ear later, its path longer by $21.5 \cos \varphi$ cm. Writing $k = \cos \varphi$, the possible directions are the lines $(1 - k^2)x^2 - k^2y^2 = 0$.

a. Which lines does a delay of 0 give? What kind of pair is it, and where can the drum be?

b. The longest possible delay is about 0.63 ms. Which lines does it give, and what kind of pair?

c. For any delay in between, how are the two lines placed with respect to the line through the ears?

8 Activity: Read a pair from its numbers (not marked)

For each equation, write a, h, b, work out $h^2 - ab$ and $a + b$, and say what the equation is. Then factorize it to check.

Equation	a, h, b	$h^2 - ab$	$a + b$	What it is	Its factors
$x^2 - 5xy + 6y^2 = 0$					
$x^2 - 4xy + 4y^2 = 0$					
$x^2 + xy + y^2 = 0$					
$2x^2 + 3xy - 2y^2 = 0$					

Which pair has the largest angle between its lines?