Unit 12 · Pair of Straight Lines

HARDER PRACTICE QUESTIONS

Full marks 26

Write your answers on separate paper. Show your working. In $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, remember that h, g and f are halves.

1 Passing the test, yet no lines [6]

Consider $x^2 + 2xy + y^2 + 2x + 2y + c = 0$.

a. Show that $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ for **every** value of c.

b. Diksha concludes that the equation is two real lines whatever c is. By writing it in terms of $u = x + y$, find for which c she is right, for which c it is one line, and what it is for the other values.

c. Find c so that the lines are 2 units apart.

2 Pairs made to order [5]

a. Show that every pair $ax^2 + 2hxy + ay^2 = 0$ (with $h^2 > a^2$) has the bisectors $y = \pm x$. Write down two different pairs with these bisectors whose lines meet at $60°$.

b. Find a pair through the origin whose bisectors are the two axes and whose lines meet at $60°$. How many such pairs are there?

3 A chord seen at a right angle [6]

The line $x + y = c$ ($c \neq 0$) cuts the circle $x^2 + y^2 - 2x - 4y - 4 = 0$ at two points P and Q.

a. At P and Q, $\dfrac{x+y}{c} = 1$. Use this to show that P and Q both lie on $x^2 + y^2 - (2x + 4y)\dfrac{x+y}{c} - 4\left(\dfrac{x+y}{c}\right)^2 = 0$, and explain why this equation is the pair of lines OP, OQ.

b. Find c so that $\angle POQ = 90°$.

c. Check, using the distance from the circle's center to the line, that the line really cuts the circle for each c you found.

4 Where is the sound? [5]

Arman's robot has two microphones on the X-axis, 20 cm apart, with the origin midway, and two more on the Y-axis, also 20 cm apart. Sound travels 34.3 cm in a millisecond, and comes from far away, so (as in the unit's drum) a path difference of $20\cos\varphi$ cm means the sound's direction makes the angle φ with that pair's axis. Round each path difference to the nearest millimetre.

a. A clap reaches the right-hand microphone (positive X) 0.350 ms before the left-hand one. Find the pair of lines the clap's direction can lie on, and say which half of each line it must be.

b. The same clap reaches the upper microphone (positive Y) 0.466 ms before the lower one. Find the pair of lines this gives, and its sign condition.

c. Where is the clap? Why can't one pair of microphones decide, when two can?

d. On another clap the readings are 0.350 ms on both pairs. Show that the robot's two pairs of lines have no line in common, and say what that tells the robot.

5 The perpendicular pair's bisectors [4]

The lines through the origin perpendicular to the lines of $ax^2 + 2hxy + by^2 = 0$ are $bx^2 - 2hxy + ay^2 = 0$ (the unit's exercise, short question 3).

a. Show that the two pairs have the same bisectors.

b. Riya checks it with $6x^2 - 5xy + y^2 = 0$. Find its perpendicular pair, both pairs' lines, and their bisectors.

c. Explain the result of a. without algebra: turn the whole picture through a right angle about O.