Unit 12 · Pair of Straight Lines

ANSWER KEY · FOR TEACHERS

Full marks 40

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 3

1. homogeneous
2. origin
3. coincident
4. imaginary
5. perpendicular
6. half

Each carries 0.5 mark.

2 True or false

U · 3

i. ✓ **True**

ii. ✗ **False**: $2h = 3$, so $h = \frac{3}{2}$.

iii. ✗ **False**: Only when $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$; $x^2 + y^2 - 4 = 0$ is a circle.

iv. ✓ **True**

v. ✗ **False**: Both pass through the origin, so they cross there or coincide.

vi. ✗ **False**: $a = b$, so $h(x^2 - y^2) = 0$: the bisectors are $x^2 - y^2 = 0$, $y = \pm x$.

Each carries 0.5 mark.

3 Multiple choice

i. **b.** $a + b = 0$ — K · 1

Then $\tan\theta = \frac{2\sqrt{h^2 - ab}}{0}$ is undefined, $\theta = 90°$: equivalently $m_1 m_2 = \frac{a}{b} = -1$. $h^2 = ab$ makes them coincide.

ii. **a.** $h^2 = ab$ — K · 1

The slopes are the roots of $bm^2 + 2hm + a = 0$, equal when its discriminant $4(h^2 - ab)$ is 0.

iii. **a.** $x = 3y$ and $x = 4y$ — U · 1

$x^2 - 7xy + 12y^2 = (x - 3y)(x - 4y)$. Check: $m_1 + m_2 = \frac{1}{3} + \frac{1}{4} = \frac{7}{12} = -\frac{2h}{b}$. Reading the factors as $y = 3x$ swaps x and y.

iv. **c.** 60° — U · 1

$h = 2$: $\tan\theta = \frac{2\sqrt{4 - 1}}{1 + 1} = \sqrt{3}$, so 60°. Taking $h = 4$ gives $\tan\theta = \sqrt{15}$, none of these.

v. **b.** 8 — A · 1

$a = 2, b = 3, h = \frac{7}{2}, g = 4, f = 7, c = k$: $6k + 196 - 98 - 48 - \frac{49}{4}k = 0$, so $\frac{25}{4}k = 50$, $k = 8$. Then it is $(2x + y + 4)(x + 3y + 2) = 0$.

vi. **a.** $8h^2 = 9ab$ — HA · 1

Slopes m and $2m$: $3m = -\frac{2h}{b}$ and $2m^2 = \frac{a}{b}$. The first gives $m = -\frac{2h}{3b}$; put in the second: $\frac{8h^2}{9b^2} = \frac{a}{b}$, $8h^2 = 9ab$. Check: $x^2 - 3xy + 2y^2 = (x - y)(x - 2y)$ has slopes 1 and $\frac{1}{2}$, and $8 \cdot \frac{9}{4} = 18 = 9 \cdot 2$. ✓

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4 Short answers

a. MARKING SCHEME U · 2

✓ The product $(y - 3x)(2x + y) = 0$.

✓ $6x^2 + xy - y^2 = 0$.

b. MARKING SCHEME U · 2

✓ $h = -3$, not -6.

✓ $\tan\theta = \frac{2}{3}$.

c. MARKING SCHEME A · 2

✓ $k = 5$.

✓ $x + 3y = 0$.

d. MARKING SCHEME A · 3

✓ $h = -5, g = \frac{11}{2}, f = -\frac{5}{2}$ (halves).

✓ $2 - k = 0, k = 2$.

✓ $h^2 - ab = 1 > 0$: real.

e. MARKING SCHEME U · 2

✓ (tx, ty) gives t^2 times the left side, still 0.

✓ These points make the line through the point and O.

5 Structured question

ANSWER A · 5

a. $a = 1, b = 4, c = -2, h = -\frac{5}{2}, g = \frac{1}{2}, f = 1$:

$abc + 2fgh - af^2 - bg^2 - ch^2 = -8 - \frac{5}{2} - 1 - 1 + \frac{25}{2} = 0$, and $h^2 - ab = \frac{9}{4} > 0$: a pair of real lines.

b. As a quadratic in x: $x^2 + (1 - 5y)x + (4y^2 + 2y - 2) = 0$, discriminant $(1 - 5y)^2 - 4(4y^2 + 2y - 2) = 9y^2 - 18y + 9 = (3y - 3)^2$. $x = \frac{5y - 1 \pm (3y - 3)}{2}$: $x = 4y - 2$ or $x = y + 1$. The lines are $x - 4y + 2 = 0$ and $x - y - 1 = 0$. $\tan\theta = \frac{2\sqrt{9/4}}{|1 + 4|} = \frac{3}{5}$, $\theta \approx 31.0°$.

MARKING SCHEME

✓ a. The values, with h, g, f halved.

✓ The condition 0 (and $h^2 > ab$).

✓ b. The discriminant $(3y - 3)^2$.

✓ $x - 4y + 2 = 0$ and $x - y - 1 = 0$.

✓ $\tan\theta = \frac{3}{5}$.

6 Long question

ANSWER

a. Let the lines be $y - m_1x = 0$ and $y - m_2x = 0$. A point of a bisector is equally far from both: $\frac{y - m_1x}{\sqrt{1 + {m_1}^2}} = \pm\frac{y - m_2x}{\sqrt{1 + {m_2}^2}}$. Squaring takes both signs at once: $(y - m_1x)^2(1 + {m_2}^2) = (y - m_2x)^2(1 + {m_1}^2)$, which collects to $({m_1}^2 - {m_2}^2)(x^2 - y^2) + 2(m_2 - m_1)(1 - m_1m_2)xy = 0$. Divide by $m_1 - m_2$: $(m_1 + m_2)(x^2 - y^2) = 2(1 - m_1m_2)xy$. With $m_1 + m_2 = -\frac{2h}{b}$, $m_1m_2 = \frac{a}{b}$, times $-\frac{b}{2}$: $h(x^2 - y^2) = (a - b)xy$. Its x^2 and y^2 coefficients, h and $-h$, add to 0: perpendicular.

b. $a = 7, h = -4, b = 1$: $-4(x^2 - y^2) = 6xy$, that is $2x^2 + 3xy - 2y^2 = 0$, $(2x - y)(x + 2y) = 0$: $y = 2x$ and $y = -\frac{1}{2}x$. The lines are $(7x - y)(x - y) = 0$, slopes 7 and 1. Between $y = 2x$ and $y = x$: $\tan\alpha = \frac{2 - 1}{1 + 2} = \frac{1}{3}$; between $y = 7x$ and $y = 2x$: $\tan\beta = \frac{7 - 2}{1 + 14} = \frac{1}{3}$. Equal.

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c. $\frac{7x-y}{5\sqrt{2}} = \pm\frac{x-y}{\sqrt{2}}$, so $7x-y = \pm 5(x-y)$: $x+2y=0$ or $2x-y=0$, the same two lines. A · 8

MARKING SCHEME

- ✓ a. Equal distances, with the ± sign.
- ✓ Squared and collected to $(m_1+m_2)(x^2-y^2) = 2(1-m_1m_2)xy$.
- ✓ $h(x^2-y^2) = (a-b)xy$.
- ✓ Perpendicular: coefficients h and $-h$.
- ✓ b. $2x^2+3xy-2y^2=0$: $y=2x$, $y=-\frac{1}{2}x$.
- ✓ Equal angles, tan $= \frac{1}{3}$ each side.
- ✓ c. $7x-y = \pm 5(x-y)$.
- ✓ $x+2y=0$ and $2x-y=0$.

7 Think

ANSWER HA · 4

a. Delay 0: $k=0$, $x^2=0$: the Y-axis counted twice ($h^2=ab$, coincident). The drum is straight ahead or straight behind, equally far from both ears.

b. The longest delay is when the path difference is the whole 21.5 cm: $k=1$, $-y^2=0$: the X-axis counted twice, coincident again. The drum is straight out to the side, on the line through the ears.

c. Here $h=0$, so the bisectors are $xy=0$, the axes: the two lines are mirror images in the line through the ears (front and back), always. Real and different for $0<k<1$, since $h^2-ab=(1-k^2)k^2>0$.

MARKING SCHEME

- ✓ a. $x^2=0$: coincident lines along the Y-axis, ahead or behind.
- ✓ b. $y^2=0$: coincident along the X-axis, to the side.
- ✓ c. $h=0$: bisectors the axes.
- ✓ The lines are mirror images in the line through the ears.

8 Activity: Read a pair from its numbers

(not marked)

WHAT STUDENTS SHOULD FIND

Row 1: 1, $-\frac{5}{2}$, 6; $\frac{1}{4}>0$; 7: two lines, $(x-2y)(x-3y)$, tan $\theta = \frac{1}{7}$, about 8.1°. Row 2: 1, −2, 4; 0: one line counted twice, $(x-2y)^2$. Row 3: 1, $\frac{1}{2}$, 1; $-\frac{3}{4}<0$: only the origin; no real factors. Row 4: 2, $\frac{3}{2}$, −2; $\frac{25}{4}$; $a+b=0$: perpendicular lines, $(2x-y)(x+2y)$, the largest angle, 90°.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	11	19	5	**40**

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