# Unit 11 · Straight Line

HARDER PRACTICE QUESTIONS

Full marks 28

Write your answers on separate paper. Show your working. Lengths are in the units of the axes.

## 1 Lines through a point, at a given distance [5]

a. Find every line through $P(2, 3)$ whose perpendicular distance from the origin is 2.

b. Prapti writes the lines as $y - 3 = m(x - 2)$ and finds only one value of $m$. Explain where her second line went.

c. For which distances $d$ are there two such lines through $P$, for which one, and for which none? Explain with a picture of the circle of radius $d$ round $O$.

## 2 The largest round tank [7]

A triangular plot of land has its three sides along $y = 3$, $4x + 3y = 49$ and $12x - 5y = 21$ (in meters, measured from a post outside the plot as the origin).

a. Find the three corners of the plot and the lengths of its sides.

b. Aakriti wants the bisector of the plot's angle at the corner $(10, 3)$. She makes both constants positive and takes the + sign, as for the angle containing the origin. Find her line, show that it is not the bisector she wants, and find the right one. Then show that the same rule does give the right bisector at the corner $(3, 3)$, and say why it works at one corner and not the other.

c. A round water tank is to be built as large as the plot allows. Find its center and radius.

d. A pipe is to come up at $(8, 6)$. On a sketch it looks inside the plot. Is it? Decide using signs, not a drawing.

## 3 Two squares on two lines [5]

Two sides of a square lie along $3x + 4y = 6$ and $6x + 8y + 8 = 0$, and one corner of the square is $(2, 0)$.

a. Find the length of the square's side.

b. Find the equations of the square's other two sides. Show that there are two such squares.

c. Find the other three corners of each square.

## 4 One pump for two villages [6]

A straight river runs along $x + y = 4$ (in km). Two villages are at $A(5, 6)$ and $B(10, 8)$. One pump on the river bank at $P$ will send water to both by straight pipes $PA$ and $PB$.

a. Show that the villages are on the same side of the river.

b. Find the foot of the perpendicular from $A$ to the river, and the point $A'$ for which that foot is the midpoint of $AA'$.

c. Explain why $PA = PA'$ for every point $P$ of the river. Use this to find where the pump should be for the least total length of pipe, and that length.

d. Rohan says: "Put the pump at the point of the river nearest to $A$." How much more pipe would that need?

## 5 Not only half and half [5]

The lines $L_1$: $3x + 4y - 7 = 0$ and $L_2$: $5x - 12y + 7 = 0$ meet at $(1, 1)$.

a. Find the set of all points whose distance from $L_1$ is twice their distance from $L_2$, as equations. What shape is it?

b. Show that the lines you found pass through $(1, 1)$, and explain why they must.

c. Are they perpendicular to each other? The bisectors (ratio 1) always are. Explain what is special about ratio 1.