Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Lines through a point, at a given distance

HA · 5

ANSWER

Tests: the two answers the size bars give; a vertical line has no slope; how far a line through P can be from O.

a. $y-3=m(x-2)$ is $mx-y+3-2m=0$: $\frac{|3-2m|}{\sqrt{m^2+1}}=2$, $9-12m+4m^2=4m^2+4$, $m=\frac{5}{12}$: $5x-12y+26=0$ ($\frac{26}{13}=2$ ✓). And the vertical line $x=2$, whose distance from O is 2. So two lines: $5x-12y+26=0$ and $x=2$.

b. The m^2 terms cancelled, so the "quadratic" in m had only one root: the other line has no slope. Every line through P except $x=2$ can be written with an m; $x=2$ must be tested on its own. (Tempting: stopping at one line.)

c. A line through P is at distance d from O when it touches the circle of radius d round O. $OP=\sqrt{13}\approx 3.61$. For $0<d<\sqrt{13}$, P is outside the circle and there are two tangents from P: two lines. For $d=\sqrt{13}$, P is on the circle: one line, perpendicular to OP, $2x+3y=13$. For $d>\sqrt{13}$, none: no line through P is farther from O than OP, since the perpendicular is the shortest path. (For $d=0$, one line, OP itself.)

MARKING SCHEME

✓ a. $\frac{|3-2m|}{\sqrt{m^2+1}}=2$, $m=\frac{5}{12}$: $5x-12y+26=0$.
✓ $x=2$ also.
✓ b. The m^2 terms cancel; the missing line is vertical, with no slope.
✓ c. Two lines for $0<d<\sqrt{13}$, one for $d=\sqrt{13}$.
✓ None for $d>\sqrt{13}$, because the distance is never more than OP.

2 The largest round tank

ANSWER

Tests: choosing a bisector from a point known to be inside, not from the origin; the incenter as the meeting of internal bisectors; the sign test. The tempting answer in b. is Aakriti's line.

a. $y=3$ with $12x-5y=21$: (3, 3); with $4x+3y=49$: (10, 3); $4x+3y=49$ with $12x-5y=21$: (5.5, 9). Sides 7, $\sqrt{4.5^2+6^2}=7.5$ and $\sqrt{2.5^2+6^2}=6.5$.

b. Constants positive: $-y+3=0$, $-4x-3y+49=0$. The + sign: $5(-y+3)=-4x-3y+49$, that is $2x-y-17=0$. The plot's angle at (10, 3) is the one holding the plot, so test with a point inside the plot, such as the centroid $G\left(\frac{37}{6}, 5\right)$ (the corners won't do: each lies on two sides): there $y-3=2>0$ and $4x+3y-49=-\frac{28}{3}<0$. The origin gives −3 and −49: it is on the plot's side of $4x+3y=49$ but not of $y=3$, so it lies in the angle **beside** the plot's angle, and Aakriti's line bisects that one, the outside angle. In the plot's angle the two values have opposite signs, so the bisector is $y-3=-\frac{4x+3y-49}{5}$: $5y-15=-4x-3y+49$, $x+2y-16=0$. At (3, 3) the rule gives $13(-y+3)=-12x+5y+21$, $2x-3y+3=0$, and that is right: the origin is on the wrong side of **both** $y=3$ and $12x-5y=21$ (values −3 and −21, against 2 and 28 at G), so it is in the vertically opposite angle, which has the same bisector. The rule finds the angle that holds the origin; whether that is the plot's angle has to be checked with a point of the plot.

c. The tank touches all three sides: its center is the incenter, where $x+2y-16=0$ meets $2x-3y+3=0$: (6, 5). Its distance from each side: $5-3=2$, $\frac{|24+15-49|}{5}=2$, $\frac{|72-25-21|}{13}=2$. Center (6, 5), radius 2 m. (Check: area $\frac{1}{2}\cdot 7\cdot 6=21$ = radius × half the perimeter $=2\times 10.5$. ✓)

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d. At (8, 6): $y - 3 = 3$ (same sign as at the inside point (6, 5), 2), $12x - 5y - 21 = 45$ (same as 26), but $4x + 3y - 49 = 1$, while at (6, 5) it is -10: the signs differ, so (8, 6) is **outside**, $\frac{1}{5} = 0.2$ m beyond the side $4x + 3y = 49$. HA · 7

MARKING SCHEME

- ✓ a. The corners (3, 3), (10, 3), (5.5, 9).
- ✓ Sides 7, 7.5, 6.5.
- ✓ b. Aakriti's line $2x - y - 17 = 0$, shown to bisect the outside angle (the origin is not in the plot's angle there).
- ✓ The right bisector $x + 2y - 16 = 0$, its sign chosen from a point inside the plot.
- ✓ At (3, 3) the rule gives $2x - 3y + 3 = 0$, right, since the origin is in the opposite angle there.
- ✓ c. Center (6, 5), radius 2 m.
- ✓ d. Outside: the sign of $4x + 3y - 49$ differs from an inside point's (0.2 m out).

3 Two squares on two lines

ANSWER A · 5

Tests: matching the coefficients before the parallel-lines formula; a perpendicular through a point; two answers from the size bars. Tempting: $\frac{|-6 - 8|}{5} = 2.8$, or only one square.

a. The second line is $3x + 4y + 4 = 0$; the first $3x + 4y - 6 = 0$. Side $= \frac{|-6 - 4|}{5} = 2$. ((2, 0) is on the first line: $6 + 0 = 6$.)

b. One side through (2, 0) is perpendicular to the two lines: $4x - 3y - 8 = 0$. The fourth side is parallel to it, 2 away: $4x - 3y + k = 0$ with $\frac{|8 + k|}{5} = 2$, so $k = 2$ or $k = -18$: $4x - 3y + 2 = 0$ or $4x - 3y - 18 = 0$, one square on each side of $4x - 3y - 8 = 0$.

c. $4x - 3y = 8$ meets $3x + 4y = -4$ at (0.8, -1.6), a corner of both squares. Square 1 (with $4x - 3y = 18$): (3.6, -1.2) and (2.4, -2.8). Square 2 (with $4x - 3y = -2$): (0.4, 1.2) and (-0.8, -0.4). Each side is 2: e.g. from (2, 0) to (3.6, -1.2), $\sqrt{2.56 + 1.44} = 2$.

MARKING SCHEME

- ✓ a. $3x + 4y + 4 = 0$ used: side 2.
- ✓ b. $4x - 3y - 8 = 0$ through (2, 0).
- ✓ $4x - 3y + 2 = 0$ and $4x - 3y - 18 = 0$: two squares.
- ✓ c. (0.8, -1.6).
- ✓ (3.6, -1.2), (2.4, -2.8) and (0.4, 1.2), (-0.8, -0.4).

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4 One pump for two villages

ANSWER HA · 6

Tests: the side test; the foot of a perpendicular from 11.1; why reflection turns a bent path into a straight one; the tempting "nearest point".

a. $x + y - 4$ is 7 at A and 14 at B: same sign, same side.

b. The perpendicular from A has slope 1: $y = x + 1$. It meets $x + y = 4$ at $N(1.5, 2.5)$. $A' = 2N - A = (-2, -1)$.

c. The river is the perpendicular bisector of AA' (it is perpendicular to AA' and passes through its midpoint N), and every point of a perpendicular bisector is equally far from the two ends: $PA = PA'$. So $PA + PB = PA' + PB$, which is least when P is on the straight segment $A'B$ (a straight line is the shortest path). $A'B$: from $(-2, -1)$ to $(10, 8)$, slope $\frac{3}{4}$, $3x - 4y + 2 = 0$; with $x + y = 4$: $P(2, 2)$. The least length is $A'B = \sqrt{144 + 81} = 15$ km ($PA = 5$, $PB = 10$).

d. At $N(1.5, 2.5)$: $NA = \sqrt{3.5^2 + 3.5^2} \approx 4.95$, $NB = \sqrt{8.5^2 + 5.5^2} \approx 10.12$, total about 15.074 km: some 74 m more. Close, but not least: the best point balances both villages, and PA and PB make equal angles with the river, like light reflected in a mirror.

MARKING SCHEME

- ✓ a. The values 7 and 14, same sign.
- ✓ b. $N(1.5, 2.5)$ and $A'(-2, -1)$.
- ✓ c. $PA = PA'$: the river is the perpendicular bisector of AA'.
- ✓ P on the segment $A'B$: $P(2, 2)$.
- ✓ Least length 15 km.
- ✓ d. About 15.074 km, some 74 m more.

5 Not only half and half

ANSWER HA · 5

Tests: the bisector derivation with a ratio other than 1; why bisectors meet at the lines' crossing; why only the bisectors are perpendicular.

a. $\frac{|3x + 4y - 7|}{5} = 2 \cdot \frac{|5x - 12y + 7|}{13}$, so $13(3x + 4y - 7) = \pm 10(5x - 12y + 7)$: $11x - 172y + 161 = 0$ or $89x - 68y - 21 = 0$. A pair of lines.

b. $11 - 172 + 161 = 0$ and $89 - 68 - 21 = 0$. They must: at $(1, 1)$ both distances are 0, and $0 = 2 \times 0$.

c. No: $11 \cdot 89 + (-172)(-68) = 12675 \neq 0$. Divided by its root, each line's equation is $\hat{a}_1 x + \hat{b}_1 y + \ldots$ with $\hat{a}_1{}^2 + \hat{b}_1{}^2 = 1$. The lines of ratio k have coefficients $(\hat{a}_1 \mp k\hat{a}_2, \hat{b}_1 \mp k\hat{b}_2)$, and the condition $A_1A_2 + B_1B_2 = 0$ becomes $1 - k^2 = 0$: perpendicular only for $k = 1$, the bisectors.

MARKING SCHEME

- ✓ a. $13(3x + 4y - 7) = \pm 10(5x - 12y + 7)$.
- ✓ $11x - 172y + 161 = 0$ and $89x - 68y - 21 = 0$.
- ✓ b. Both pass through $(1, 1)$, where both distances are 0.
- ✓ c. Not perpendicular ($12675 \neq 0$).
- ✓ $A_1A_2 + B_1B_2 = 1 - k^2$, zero only for $k = 1$.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	0	5	23	**28**

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