Unit 11 · Straight Line

ANSWER KEY · FOR TEACHERS

Full marks 40

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 3

1. perpendicular
2. foot
3. same side
4. bisector
5. origin
6. same

Each carries 0.5 mark.

2 True or false

U · 3

i. ✓ **True**

ii. ✗ **False**: It is $\frac{|3-4+5|}{\sqrt{3^2+4^2}} = \frac{4}{5}$: divide by the square root.

iii. ✓ **True**

iv. ✗ **False**: It gives the bisector of the angle containing the origin, which may be the obtuse one; test the angle ($\tan\theta < 1$) to find the acute bisector.

v. ✓ **True**

vi. ✗ **False**: $6 + 4 - 8 = 2 > 0$ but $c = -8 < 0$: the signs differ, so it is on the other side.

Each carries 0.5 mark.

3 Multiple choice

i. K · 1

b. $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

The size of the line's left side at the point, over $\sqrt{a^2+b^2}$. The first is the t of the derivation, with its sign; the last is the distance to the origin, not to the line.

ii. K · 1

b. perpendicular to each other

They bisect two angles that add up to 180°, so the angle between them is 90°.

iii. U · 1

b. $\frac{3}{2}$

Halve the second: $3x + 4y + \frac{5}{2} = 0$. Then $\frac{|-5-5/2|}{5} = \frac{3}{2}$. 2 comes from $\frac{|-5-5|}{5}$, without halving. They are parallel, so not 0.

iv. U · 1

b. $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Write it $\frac{1}{a}x + \frac{1}{b}y - 1 = 0$: $p = \frac{1}{\sqrt{1/a^2 + 1/b^2}}$, so $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$. $a^2 + b^2$ is the square of the hypotenuse between the intercepts, not of the altitude.

v. A · 1

b. $-\frac{3}{2}$ or $\frac{7}{2}$

$\frac{|3-4k+1|}{5} = 2$, so $4 - 4k = \pm 10$: $k = -\frac{3}{2}$ or $k = \frac{7}{2}$. The size bars give two points, one on each side of the line.

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vi. **a.** -2 HA · 1

As $2x - y + c = 0$: $\frac{|c|}{\sqrt{5}} = \frac{|4 + c|}{\sqrt{5}}$, so $c = -(4 + c)$ and $c = -2$ ($c = 4 + c$ is impossible). The line passes through the midpoint $(1, 0)$: a line not parallel to AB is equally far from A and B only if it cuts AB in half.

4 Short answers

a. MARKING SCHEME U · 2

✓ $|10 + 36 + 6| = 52$ and $\sqrt{169} = 13$.

✓ 4.

b. MARKING SCHEME U · 2

✓ The equations not made to match (halve the second).

✓ $\sqrt{5}$.

c. MARKING SCHEME A · 2

✓ $\frac{|4h - 12|}{5} = 4$.

✓ $(8, 0)$ and $(-2, 0)$.

d. MARKING SCHEME A · 3

✓ A parallel line written $3x - 4y + k = 0$.

✓ $\frac{|k - 1|}{5} = 2$.

✓ $3x - 4y + 11 = 0$ and $3x - 4y - 9 = 0$.

e. MARKING SCHEME U · 2

✓ The right angle at N: PQ is the hypotenuse.

✓ $PQ^2 = PN^2 + NQ^2 > PN^2$.

5 Structured question

ANSWER A · 5

a. $\sqrt{16 + 9} = 5$, $\sqrt{576 + 49} = 25$: $\frac{4x + 3y - 7}{5} = \pm\frac{24x + 7y - 31}{25}$, that is $5(4x + 3y - 7) = \pm(24x + 7y - 31)$. The + sign gives $-4x + 8y - 4 = 0$, $x - 2y + 1 = 0$; the − sign $44x + 22y - 66 = 0$, $2x + y - 3 = 0$.

b. Between $2x + y - 3 = 0$ (slope -2) and the first line (slope $-\frac{4}{3}$): $\tan\theta = \left|\frac{-2 + \frac{4}{3}}{1 + \frac{8}{3}}\right| = \frac{2}{11} < 1$.

So $2x + y - 3 = 0$ bisects the acute angle.

c. With the constants positive, $-4x - 3y + 7 = 0$ and $-24x - 7y + 31 = 0$, the + sign gives $5(-4x - 3y + 7) = -24x - 7y + 31$, that is $x - 2y + 1 = 0$: the origin is in the obtuse angle.

MARKING SCHEME

✓ a. $5(4x + 3y - 7) = \pm(24x + 7y - 31)$.

✓ $x - 2y + 1 = 0$ and $2x + y - 3 = 0$.

✓ b. $\tan\theta = \frac{2}{11}$ (or $\tan\theta > 1$ for the other).

✓ $2x + y - 3 = 0$ bisects the acute angle.

✓ c. $x - 2y + 1 = 0$.

6 Long question

ANSWER

a. $\frac{|0 + 0 - 120|}{\sqrt{144 + 25}} = \frac{120}{13}$.

b. The bisectors at A, between $y = 0$ and $12x + 5y - 120 = 0$, are $y = \pm\frac{12x + 5y - 120}{13}$. Inside the triangle $y > 0$ and $12x + 5y - 120 < 0$ (the origin's side), so the inside bisector is $13y = -(12x + 5y - 120)$, $12x + 18y - 120 = 0$: $2x + 3y - 20 = 0$. (The other sign gives $3x - 2y - 30 = 0$, the bisector of the outside angle.)

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c. At O, between the axes inside the first quadrant: $y = x$. At $B(0, 24)$, the same way: $13x = -(12x + 5y - 120)$, $5x + y - 24 = 0$. $y = x$ and $2x + 3y - 20 = 0$ meet where $5x = 20$: $(4, 4)$, and $20 + 4 - 24 = 0$, so it is on the third bisector too. It is 4 from $x = 0$, 4 from $y = 0$, and $\frac{|48 + 20 - 120|}{13} = 4$ from the third side: the incenter, the center of the circle of radius 4 touching all three sides. **A · 8**

MARKING SCHEME

- ✓ a. The formula with $(0, 0)$.
- ✓ $\frac{120}{13}$.
- ✓ b. $y = \pm\frac{12x + 5y - 120}{13}$.
- ✓ The sign chosen from the inside of the triangle.
- ✓ $2x + 3y - 20 = 0$.
- ✓ c. The bisectors $y = x$ and $5x + y - 24 = 0$.
- ✓ They and $2x + 3y - 20 = 0$ all pass through $(4, 4)$.
- ✓ 4 from each side.

7 Think

ANSWER **HA · 4**

Equally far from the axes: $|x| = |y|$, so the point is on $y = x$ or $y = -x$. Its distance from the third line is $\frac{|x + y - 2|}{\sqrt{2}}$.

On $y = x$: $\sqrt{2}\,|x| = |2x - 2|$, so $x = 2 - \sqrt{2}$ or $x = 2 + \sqrt{2}$.

On $y = -x$: $|x| = \frac{2}{\sqrt{2}} = \sqrt{2}$, so $x = \pm\sqrt{2}$.

Four points: $(2 - \sqrt{2}, 2 - \sqrt{2})$, $(2 + \sqrt{2}, 2 + \sqrt{2})$, $(\sqrt{2}, -\sqrt{2})$, $(-\sqrt{2}, \sqrt{2})$. Inside the triangle $(x, y > 0, x + y < 2)$: $(2 - \sqrt{2}, 2 - \sqrt{2}) \approx (0.59, 0.59)$, the incenter. The other three are outside, each in one of the outside angles.

MARKING SCHEME

- ✓ Equally far from the axes: on $y = \pm x$.
- ✓ On $y = x$: $(2 \mp \sqrt{2}, 2 \mp \sqrt{2})$.
- ✓ On $y = -x$: $(\sqrt{2}, -\sqrt{2})$ and $(-\sqrt{2}, \sqrt{2})$.
- ✓ Inside: $(2 - \sqrt{2}, 2 - \sqrt{2})$.

8 Activity: Measure, then compute

(not marked)

WHAT STUDENTS SHOULD FIND

Values -12, 12, -5, 2; distances $\frac{12}{5} = 2.4$, 2.4, 1 and 0.4 cm, the measured ones within a millimetre or two. $(0, 0)$ and $(4, 3)$ are both 2.4 from the line but on opposite sides (their values have opposite signs). $(1, 1)$ is on the origin's side, $(6, -1)$ on the other.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	11	19	5	**40**

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