# Unit 10 · Trigonometric Equations and General Values

WORKSHEET

Full marks 40

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 40

## 1 Key words [3]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

| | | | | | | |
|---|---|---|---|---|---|---|
| **amplitude** | **general solution** | **homogeneous** | **identity** | **inverse** | **periodic** | **principal value** |
| **trigonometric equation** | | | | | | |

$\sin^2\theta + \cos^2\theta = 1$, true for every angle, is an _____; $2\cos\theta = 1$, true only for some angles, is a _____. One formula with $n \in \mathbb{Z}$ that gives every solution is the _____. The usual choice of $\alpha$ in it is the _____ of Unit 9. The sine and cosine repeat every $2\pi$ and the tangent every $\pi$: each is a _____ function. An equation in which every term has degree 2 in $\sin x$ and $\cos x$ is _____, and becomes a quadratic in $\tan x$ after dividing by $\cos^2 x$.

## 2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. The general solution of $\sin\theta = \frac{1}{2}$ is $\theta = 2n\pi \pm \frac{\pi}{6}$. ☐

ii. The general solution of $\sin^2\theta = \frac{1}{4}$ is $\theta = n\pi \pm \frac{\pi}{6}$. ☐

iii. Dividing $\sin 2x = \cos x$ by $\cos x$ gives all its solutions. ☐

iv. Every trigonometric equation has infinitely many solutions. ☐

v. The general solution of $\cos\theta = 1$ is $\theta = n\pi$. ☐

vi. The general solution of $\tan\theta = 3$ is $\theta = n\pi + \tan^{-1} 3$. ☐

## 3 Multiple choice [6]

Choose the correct option for each of the following questions.

**i.** The general solution of $\sin\theta = 0$ is: [1]

**a.** $\theta = n\pi$ **b.** $\theta = 2n\pi$

**c.** $\theta = (2n+1)\frac{\pi}{2}$ **d.** $\theta = \frac{n\pi}{2}$

**ii.** If $\tan\theta = \tan\alpha$, then: [1]

**a.** $\theta = 2n\pi \pm \alpha$ **b.** $\theta = n\pi + (-1)^n\alpha$

**c.** $\theta = n\pi + \alpha$ **d.** $\theta = n\pi \pm \alpha$

**iii.** The general solution of $\cos\theta = -\frac{1}{2}$ is: [1]

**a.** $\theta = 2n\pi \pm \frac{2\pi}{3}$ **b.** $\theta = 2n\pi \pm \frac{\pi}{3}$

**c.** $\theta = n\pi \pm \frac{2\pi}{3}$ **d.** $\theta = n\pi + (-1)^n \frac{2\pi}{3}$

**iv.** The general solution of $\tan^2\theta = 3$ is: [1]

**a.** $\theta = n\pi + \frac{\pi}{3}$ **b.** $\theta = n\pi \pm \frac{\pi}{3}$

**c.** $\theta = 2n\pi \pm \frac{\pi}{3}$ **d.** $\theta = n\pi \pm \frac{\pi}{6}$

**v.** The solutions of $2\cos^2 x + \cos x = 0$ in $0 \leq x \leq \pi$ are: [1]

**a.** $\frac{\pi}{2}$ and $\frac{\pi}{3}$

**b.** $\frac{\pi}{2}$ and $\frac{2\pi}{3}$

**c.** $\frac{2\pi}{3}$ only

**d.** $\frac{\pi}{2}$, $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$

**vi.** $\sin^4 x + \cos^4 x = k$ has a solution exactly when: [1]

**a.** $0 \leq k \leq 1$ **b.** $\frac{1}{2} \leq k \leq 1$

**c.** $-1 \leq k \leq 1$ **d.** $\frac{1}{4} \leq k \leq 1$

## 4 Short answers [12]

Write short answers to the following questions.

**a.** Write the general solutions of $\sin x = 1$, $\cos x = 0$ and $\tan x = 0$. [3]

**b.** Find the general value of $\theta$ if $\sqrt{3}\sec\theta = -2$. [2]


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**c.** Simran solved $\cos^2 x = \frac{1}{4}$ like this: "$\cos x = \frac{1}{2} = \cos \frac{\pi}{3}$, so $x = 2n\pi \pm \frac{\pi}{3}$." What did she miss? Write the complete general solution. **[2]**

**d.** Tenzing says that $x = n\pi \pm \frac{\pi}{3}$ is the general solution of $\cos x = \frac{1}{2}$. Show that he is wrong, and write an equation that it really solves. Which of its solutions lie in $0 \leq x \leq \pi$? **[2]**

**e.** Solve $\sin 2x = \cos x$ for $0 \leq x \leq 2\pi$. **[3]**

## 5 Structured question **[5]**

Answer all the parts.

a. Write $\sin x + \sqrt{3} \cos x$ in the form $r \sin (x + \gamma)$. For which values of $k$ has $\sin x + \sqrt{3} \cos x = k$ a solution? [2]

b. Solve $\sin x + \sqrt{3} \cos x = \sqrt{2}$, and give its solutions in $0 \leq x < 2\pi$. [3]


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## 6 Long question [8]

Write a long answer to the following question.

a. Prove that the general solution of $\cos\theta = \cos\alpha$ is $\theta = 2n\pi \pm \alpha$. [3]

b. Solve $\cos\theta + \cos 7\theta = \cos 4\theta$. [3]

c. Find all its solutions in $0 \leq \theta \leq \frac{\pi}{2}$. [2]

## 7 Think [3]

For which values of $k$ has $\sin^2 x - \sin x = k$ a solution? Rabina says "for $-1 \leq k \leq 1$, like every sine equation". Is she right?

## 8 Activity: Two patterns side by side

(not marked)

Work out each angle (as a multiple of $\pi$) and its sine. Use $n\pi + (-1)^n \frac{\pi}{6}$ in its column and $2n\pi \pm \frac{\pi}{6}$, with each sign, in the last two.

| $n$ | $n\pi + (-1)^n \frac{\pi}{6}$ and its sine | $2n\pi + \frac{\pi}{6}$ and its sine | $2n\pi - \frac{\pi}{6}$ and its sine |
|---|---|---|---|
| $-1$ | | | |
| 0 | | | |
| 1 | | | |
| 2 | | | |

Which pattern gives the solutions of $\sin\theta = \frac{1}{2}$? Which equation does the other pattern solve?