Unit 10 · Trigonometric Equations and General Values

HARDER PRACTICE QUESTIONS

Full marks 31

Write your answers on separate paper. Show your working. Give every general solution with $n \in \mathbb{Z}$, and check every family in the equation you were given.

1 When the rule gives too much [5]

a. Ujwal solves $\tan 3x = \tan x$ by the rule for $\tan\theta = \tan\alpha$ and writes $x = \frac{n\pi}{2}$. Test his answer, and give the correct general solution.

b. Solve $\tan 2x \tan x = 1$. Write your answer as simply as you can.

c. In both a. and b. a correct rule gave values that are not solutions. Say in one sentence why the rule for $\tan\theta = \tan\alpha$ can do that, and what you must check every time you use it.

2 A constant on the right [5]

a. Sujan solves $\sin^2 x + \sin x \cos x = 1$ by dividing by $\cos^2 x$: "$\tan^2 x + \tan x = \sec^2 x = 1 + \tan^2 x$, so $\tan x = 1$." Find the solutions in $0 \le x \le 2\pi$ correctly, and say how many he lost.

b. Solve $\sin^2 x + 2\sin x \cos x = 1$ without dividing.

c. Show that $a\sin^2 x + b\sin x \cos x + c\cos^2 x = k$ has $x = \frac{\pi}{2}$ among its solutions exactly when $a = k$. So when is dividing by $\cos^2 x$ safe?

3 Sum and product together [5]

a. Solve $\sin x + \cos x = 1 + \sin x \cos x$.

b. Find the greatest and least values of $\sin x + \cos x + \sin x \cos x$, and one x where each is reached.

4 How many solutions? [6]

Consider $\cos 2x + 2\sin x = k$ for $0 \le x < 2\pi$.

a. Write it as a quadratic in $t = \sin x$. For which k has it any solution?

b. How many solutions has it in $0 \le x < 2\pi$ when $k = \frac{3}{2}$, when $k = 1$, when $k = 0$, and when $k = -3$? Find them for $k = \frac{3}{2}$ and $k = 1$.

c. For which k are there exactly four solutions?

5 Two wires of a three-phase supply [6]

In a three-phase supply the voltages of two wires are $v_1 = 325\sin(100\pi t)$ and $v_2 = 325\sin\left(100\pi t - \frac{2\pi}{3}\right)$ volts, t in seconds.

a. Find every instant in $0 \le t \le 0.02$ when $v_1 = v_2$. Splitting n into even and odd, explain what happens to one of the two cases.

b. What is the voltage of each wire at those instants?

c. Show that $v_1 - v_2$ is itself one cosine wave, and find its peak. For what fraction of each cycle is $v_1 > v_2$?

6 Make your own [4]

a. Write an equation whose solutions in $0 \leq x \leq 2\pi$ are exactly $\frac{\pi}{6}$, $\frac{\pi}{2}$ and $\frac{5\pi}{6}$. Check that it has no others there.

b. Swastika tries to find an equation of the form $a \cos x + b \sin x = c$ with the same three solutions. Explain why she can't.