Unit 9 · Inverse Circular Functions

WORKSHEET

Full marks 40

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 40

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

arcsin	**asymptotes**	**branch**	**cosecant**	**domain**	**mirror image**	**one-to-one**	**period**
principal value	**range**						

The sine is not _____, so it has an inverse only on a restricted domain. The one angle $\sin^{-1} x$ gives is its _____, and the restricted range is the principal value _____. The _____ of $\cos^{-1} x$ is $[-1, 1]$ and its _____ is $[0, \pi]$. The graph of an inverse circular function is the _____ of its function's branch in the line $y = x$. $y = \tan^{-1} x$ has the horizontal _____ $y = \pm\frac{\pi}{2}$. Many books and programming languages write $\sin^{-1} x$ as _____ x.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. $\sin^{-1} x = \dfrac{1}{\sin x}$. ☐

ii. $\cos^{-1}(-x) = \pi - \cos^{-1} x$ for $-1 \le x \le 1$. ☐

iii. $\sin^{-1}(\sin\theta) = \theta$ for every angle θ. ☐

iv. $\tan^{-1} x + \cot^{-1} x = \dfrac{\pi}{2}$ for every real x. ☐

v. $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\dfrac{x+y}{1-xy}$ for all x, y. ☐

vi. $\tan^{-1} x$ is defined for every real number x. ☐

3 Multiple choice [6]

Choose the correct option for each of the following questions.

i. The domain of $\cos^{-1} x$ is: [1]

a. $[0, \pi]$ **b.** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

c. $[-1, 1]$ **d.** all real numbers

ii. The principal values of $\tan^{-1} x$ lie in: [1]

a. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **b.** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

c. $(0, \pi)$ **d.** $[0, \pi]$

iii. $\cot^{-1}(-1)$ is: [1]

a. $-\frac{\pi}{4}$ **b.** $\frac{\pi}{4}$

c. $\frac{5\pi}{4}$ **d.** $\frac{3\pi}{4}$

iv. $\tan\left(\sin^{-1}\left(-\frac{5}{13}\right)\right)$ equals: [1]

a. $\frac{5}{12}$ **b.** $-\frac{5}{12}$

c. $-\frac{12}{5}$ **d.** $-\frac{5}{13}$

v. If $\tan^{-1} x + \tan^{-1} 3 = \frac{\pi}{4}$, then x is: [1]

a. $\frac{1}{3}$ **b.** 2

c. $-\frac{1}{2}$ **d.** -2

vi. $\sin^{-1}(\sin 10)$, with 10 in radians, equals: [1]

a. $3\pi - 10$ **b.** 10

c. $10 - 3\pi$ **d.** $10 - 2\pi$

4 Short answers [11]

Write short answers to the following questions.

a. Find the principal values of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ and $\tan^{-1}(-\sqrt{3})$. [2]

b. Aman writes: "$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \frac{7\pi}{6}$, since $\cos^{-1}$ undoes cos ." Find his mistake and give the right value. [2]

c. Find $\sin\left(\cos^{-1}\left(-\frac{5}{13}\right)\right)$ without a calculator, and explain its sign. [2]

d. If $\cos^{-1} x = \frac{2\pi}{3}$, find x and $\sin^{-1} x$, and check them with $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$. **[2]**

e. **[3]**

a. Prove that $\tan^{-1} \frac{1-x}{1+x} = \frac{\pi}{4} - \tan^{-1} x$ for $x > -1$. [2]

b. Hence find $\tan^{-1} \frac{1-\sqrt{3}}{1+\sqrt{3}}$. [1]

5 Structured question **[5]**

Answer all the parts.

$\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$.

a. Take the tangent of both sides and find the values of x it gives. [3]

b. Check each value in the original equation, and say why a check is needed. [2]

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https://science-playground.com/learn/grade-11-math/inverse-circular-functions

6 Long question [8]

Write a long answer to the following question.

a. Prove that $2\tan^{-1} x = \cos^{-1}\dfrac{1-x^2}{1+x^2}$ for $x \geq 0$, saying where you use $x \geq 0$. [3]

b. Solve $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$. [3]

c. Show that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$. [2]

7 Think [3]

Bishal uses $2\tan^{-1} x = \sin^{-1}\dfrac{2x}{1+x^2}$ at $x = 2$. His calculator gives $2\tan^{-1} 2 \approx 2.214$ but $\sin^{-1}\dfrac{4}{5} \approx 0.927$. Explain why they differ, and how the two numbers are related.

8 Activity: One principal value each

(not marked)

Use a calculator in degree mode. For each value of x, find $\sin^{-1} x$ and $\cos^{-1} x$, and add them. Then write one more angle that has the same sine as x.

x	$\sin^{-1} x$	$\cos^{-1} x$	Their sum	Another angle with sine x
0.5				
0.8				
−0.5				
1				
−1				

Why does the calculator never give the angle in your last column?