Unit 8 · Complex Number

HARDER PRACTICE QUESTIONS

Full marks 25

Write your answers on separate paper. Show your working. $z = x + yi$ with x, y real wherever you need it.

1 An unknown inside the modulus [4]

a. Find the complex number z with $z + |z| = 2 + 8i$.

b. Anish starts: "$|z|$ is complex, so I can't compare parts." What is wrong with his start?

2 Roots that are not a pair [5]

a. Find the square roots of $-3 + 4i$.

b. Solve $z^2 - 3z + (3 - i) = 0$, and check your roots with their sum and product.

c. Shreya says: "A quadratic's non-real roots always come as a conjugate pair, so your roots must be wrong." Explain why they are right anyway, and say when Shreya's rule does hold.

3 On the unit circle [5]

a. Without multiplying out, find $\left| \dfrac{(3+4i)^5(1-i)^4}{(5-12i)^2} \right|$.

b. Show that if $|z| = 1$, then $\dfrac{1}{z} = \bar{z}$.

c. Hence show that if $|z| = 1$ and $z \neq -1$, then $w = \dfrac{z-1}{z+1}$ is purely imaginary (or 0). Check with $z = \dfrac{3}{5} + \dfrac{4}{5}i$.

4 A line, or not? [4]

Pratik says: "$|z - 1| = |z + 1|$ is the imaginary axis, so $|z - 2| = 2|z + 1|$ must be a straight line too."

a. Find the set of points z with $|z - 2| = 2|z + 1|$, and describe it.

b. Check your answer with two points of it.

5 A square from one side [4]

On the Argand plane, $A = 1 + 2i$ and $B = 4 + 3i$ are two neighboring corners of a square $ABCD$.

a. Explain why $C = B + i(B - A)$ and $D = A + i(B - A)$ give the other two corners, and find them.

b. Sabina finds different corners. Could she also be right? Find every possible square.

6 A weighted sum of powers [3]

Rojina remembers that $i + i^2 + \ldots + i^{100} = 0$, so she says $1 \cdot i + 2i^2 + 3i^3 + \ldots + 100i^{100} = 0$ too. Find its true value.