Unit 8 · Complex Number

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 25

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 An unknown inside the modulus

ANSWER A · 4

Tests: one complex equation is two real ones, even when $|z|$ is in it.

a. With $z = x + yi$: $x + \sqrt{x^2 + y^2} + yi = 2 + 8i$. Imaginary parts: $y = 8$. Real parts: $x + \sqrt{x^2 + 64} = 2$, so $\sqrt{x^2 + 64} = 2 - x$; squaring, $x^2 + 64 = 4 - 4x + x^2$, $x = -15$ (and $2 - x = 17 > 0$, so the squaring added nothing false). $z = -15 + 8i$. Check: $|z| = 17$, $-15 + 8i + 17 = 2 + 8i$. ✓

b. $|z| = \sqrt{x^2 + y^2}$ is a real number, so it adds only to the real part. Comparing parts is exactly the way in.

MARKING SCHEME

✓ a. Imaginary parts: $y = 8$.
✓ Real parts: $x + \sqrt{x^2 + 64} = 2$.
✓ $x = -15$, $z = -15 + 8i$, checked.
✓ b. $|z|$ is real: it belongs to the real part.

2 Roots that are not a pair

ANSWER HA · 5

Tests: the square root with the sign rule inside the quadratic formula; the conjugate-pair rule needs real coefficients.

a. $|-3 + 4i| = 5$: $x^2 = \dfrac{5 - 3}{2} = 1$, $y^2 = \dfrac{5 + 3}{2} = 4$, same signs since $4 > 0$: $\pm(1 + 2i)$.

b. $D = 9 - 4(3 - i) = -3 + 4i$, so $z = \dfrac{3 \pm (1 + 2i)}{2}$: $z = 2 + i$ or $z = 1 - i$. Sum 3 ✓; product $(2 + i)(1 - i) = 2 - 2i + i + 1 = 3 - i$ ✓.

c. The rule comes from taking conjugates of the whole equation: if z is a root of an equation with **real** coefficients, $\bar{z}$ is too. Here the constant $3 - i$ is not real, so the conjugate of the equation is a different equation, and $2 - i$ is not a root $((2 - i)^2 - 3(2 - i) + 3 - i = 3 - 4i - 6 + 3i + 3 - i = -2i \neq 0)$.

MARKING SCHEME

✓ a. $\pm(1 + 2i)$.
✓ b. $D = -3 + 4i$.
✓ The roots $2 + i$ and $1 - i$.
✓ Checked by sum and product.
✓ c. The pair rule needs real coefficients; here one is not real.

3 On the unit circle

ANSWER

Tests: the modulus of products and quotients; $z\bar{z} = |z|^2$ used instead of expanding; "purely imaginary" as $\bar{w} = -w$.

a. $\dfrac{|3 + 4i|^5 |1 - i|^4}{|5 - 12i|^2} = \dfrac{5^5 \times (\sqrt{2})^4}{13^2} = \dfrac{3125 \times 4}{169} = \dfrac{12500}{169}$.

b. $z\bar{z} = |z|^2 = 1$, so $\bar{z} = \dfrac{1}{z}$.

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c. HA · 5

$\bar{w} = \dfrac{\bar{z}-1}{\bar{z}+1} = \dfrac{\dfrac{1}{z}-1}{\dfrac{1}{z}+1} = \dfrac{1-z}{1+z} = -w$. A number with $\bar{w} = -w$ has real part 0: w is purely imaginary (or 0, at $z = 1$). Check: $w = \dfrac{-\frac{2}{5}+\frac{4}{5}i}{\frac{8}{5}+\frac{4}{5}i} = \dfrac{-2+4i}{8+4i} = \dfrac{(-2+4i)(8-4i)}{80} = \dfrac{40i}{80} = \dfrac{1}{2}i$.

✓

MARKING SCHEME

✓ a. $\dfrac{5^5 \times 4}{13^2} = \dfrac{12500}{169}$.

✓ b. $z\bar{z} = 1$ gives $\bar{z} = \dfrac{1}{z}$.

✓ c. $\bar{w}$ written with $\dfrac{1}{z}$.

✓ $\bar{w} = -w$, so the real part is 0.

✓ The check gives $\dfrac{1}{2}i$.

4 A line, or not?

ANSWER HA · 4

Tests: reading moduli as distances; the equation, not the look, decides the shape.

a. $(x-2)^2 + y^2 = 4\left[(x+1)^2 + y^2\right]$, so $x^2 - 4x + 4 + y^2 = 4x^2 + 8x + 4 + 4y^2$, that is $3x^2 + 3y^2 + 12x = 0$, or $x^2 + y^2 + 4x = 0$: $(x+2)^2 + y^2 = 4$. A **circle** with center -2 (the point $(-2, 0)$) and radius 2. Only when the two distances are equal (ratio 1) do the squares cancel to leave a line.

b. $z = 0$: $|0-2| = 2$, $2|0+1| = 2$. ✓ $z = -4$: 6 and $2 \times 3 = 6$. ✓ (Or $z = -2 + 2i$: both $2\sqrt{5}$.)

MARKING SCHEME

✓ a. Both sides squared with $z = x + yi$.

✓ $(x+2)^2 + y^2 = 4$.

✓ A circle, center $(-2, 0)$, radius 2; a line only for ratio 1.

✓ b. Two points checked.

5 A square from one side

ANSWER HA · 4

Tests: $B - A$ as the arrow from A to B; multiplying by i as a quarter turn; the second square (by $-i$).

a. $B - A = 3 + i$ is the side AB as an arrow. Multiplying by i turns it 90° without changing its length: $i(3+i) = -1 + 3i$ is a side at right angles to AB, of the same length. Adding it to B and A gives $C = 3 + 6i$ and $D = 5i$.

b. Yes: turning the other way, by $-i$ ($-i(3+i) = 1 - 3i$), gives the square on the other side of AB: $C = 5$, $D = 2 - i$. So there are two squares, $1 + 2i$, $4 + 3i$, $3 + 6i$, $5i$ and $1 + 2i$, $4 + 3i$, 5, $2 - i$.

MARKING SCHEME

✓ a. $B - A$ is the side; times i: turned 90°, same length.

✓ $C = 3 + 6i$, $D = 5i$.

✓ b. The other square, by $-i$.

✓ $C = 5$, $D = 2 - i$.

6 A weighted sum of powers

MARKING SCHEME A · 3

✓ The blocks of four, with i, -1, $-i$, 1.

✓ Each block $2 - 2i$.

✓ $50 - 50i$.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	0	7	18	**25**

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