Unit 8 · Complex Number

ANSWER KEY · FOR TEACHERS

Full marks 43

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 4

1. imaginary unit
2. real part
3. imaginary part
4. purely imaginary
5. conjugate
6. modulus
7. Argand
8. triangle inequality

Each carries 0.5 mark.

2 True or false

U · 3

i. ✓ **True**

ii. ✗ **False**: $\sqrt{-4} \times \sqrt{-9} = 2i \times 3i = 6i^2 = -6$: $\sqrt{a}\sqrt{b} = \sqrt{ab}$ fails when both are negative.

iii. ✗ **False**: It is the real number -5.

iv. ✓ **True**

v. ✗ **False**: Complex numbers have no order: an inequality between them means nothing.

vi. ✓ **True**

Each carries 0.5 mark.

3 Multiple choice

i. **d.** $\sqrt{a^2 + b^2}$ — K · 1

The distance of (a, b) from O, by Pythagoras. $a^2 + b^2$ is its square, $z\bar{z}$.

ii. **a.** $-i$ — U · 1

$51 = 4 \times 12 + 3$, so $i^{51} = i^3 = -i$.

iii. **c.** i — U · 1

$\frac{(2 + 3i)(3 + 2i)}{9 + 4} = \frac{13i}{13} = i$. $\frac{12 + 5i}{13}$ multiplies top and bottom by $3 - 2i$ itself instead of its conjugate.

iv. **b.** 5 — A · 1

$|(1 + 2i) - (4 - 2i)| = |-3 + 4i| = 5$. 7 adds the two differences instead of using Pythagoras.

v. **d.** $\pm(1 + 2i)$ — A · 1

$|-3 + 4i| = 5$: $x^2 = \frac{5 - 3}{2} = 1$, $y^2 = \frac{5 + 3}{2} = 4$, same signs since $4 > 0$. $(2 + i)^2 = 3 + 4i$, the root of a different number.

vi. **b.** 8 — HA · 1

$(1 + i)^2 = 2i$, so $(1 + i)^4 = -4$ (real but negative) and $(1 + i)^8 = 16$. $n = 6$ gives $-8i$; odd powers are never real.

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4 Short answers

a. MARKING SCHEME U · 2

✓ The rule $\sqrt{a}\sqrt{b} = \sqrt{ab}$ fails for two negatives.

✓ $4i \times 5i = -20$.

b. MARKING SCHEME U · 2

✓ $i^{-3} = i$.

✓ The sum is 0 (25 blocks of four).

c. MARKING SCHEME A · 2

✓ Top and bottom multiplied by the conjugate $2 + i$.

✓ $1 + 2i$.

d. MARKING SCHEME A · 2

✓ $w = \dfrac{5 + 5i}{2 - i}$, divided by the conjugate.

✓ $w = 1 + 3i$, checked.

e. MARKING SCHEME A · 3

✓ $(2 + i)^2 = 3 + 4i$.

✓ $3x - 2yi = \dfrac{14 - 2i}{5}$.

✓ Parts equated: $x = \dfrac{14}{15}$, $y = \dfrac{1}{5}$.

f. MARKING SCHEME U · 3

✓ $\overline{z_1 z_2}$ worked out.

✓ $\bar{z}_1 \bar{z}_2$ worked out, the same.

✓ The check: both $5 + 5i$.

5 Structured question

ANSWER A · 5

a. $|7 - 24i| = 25$. $x^2 = \dfrac{25 + 7}{2} = 16$, $y^2 = \dfrac{25 - 7}{2} = 9$; $b = -24 < 0$: opposite signs. $\pm(4 - 3i)$.

b. $D = (2 + i)^2 - 4(-1 + 7i) = 7 - 24i$, so $x = \dfrac{(2 + i) \pm (4 - 3i)}{2}$: $x = 3 - i$ or $x = -1 + 2i$. Check: sum $2 + i$, product $(3 - i)(-1 + 2i) = -1 + 7i$. ✓

MARKING SCHEME

✓ $x^2 = 16$, $y^2 = 9$.

✓ Opposite signs: $\pm(4 - 3i)$.

✓ $D = 7 - 24i$.

✓ The formula with $\sqrt{D} = \pm(4 - 3i)$.

✓ $3 - i$ and $-1 + 2i$.

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6 Long question

ANSWER A · 8

a. $|z| = \sqrt{1+3} = 2$; $\bar{z} = 1 - \sqrt{3}\,i$; $\dfrac{1}{z} = \dfrac{\bar{z}}{z\bar{z}} = \dfrac{1-\sqrt{3}\,i}{4}$.

b. $z^2 = 1 + 2\sqrt{3}\,i + 3i^2 = -2 + 2\sqrt{3}\,i$; $z^3 = (-2 + 2\sqrt{3}\,i)(1 + \sqrt{3}\,i) = -2 - 2\sqrt{3}\,i + 2\sqrt{3}\,i + 6i^2 = -8$.

c. $x^2 = \dfrac{2+1}{2} = \dfrac{3}{2}$, $y^2 = \dfrac{2-1}{2} = \dfrac{1}{2}$, same signs: $\pm\dfrac{\sqrt{3}+i}{\sqrt{2}}$. Check: $\dfrac{3 + 2\sqrt{3}\,i - 1}{2} = 1 + \sqrt{3}\,i$. ✓

MARKING SCHEME

- ✓ a. $|z| = 2$.
- ✓ $\bar{z} = 1 - \sqrt{3}\,i$.
- ✓ $\dfrac{1}{z} = \dfrac{1-\sqrt{3}\,i}{4}$.
- ✓ b. $z^2 = -2 + 2\sqrt{3}\,i$.
- ✓ The product for z^3 multiplied out.
- ✓ $z^3 = -8$.
- ✓ c. $x^2 = \dfrac{3}{2}$, $y^2 = \dfrac{1}{2}$, same signs.
- ✓ $\pm\dfrac{\sqrt{3}+i}{\sqrt{2}}$.

7 Think

ANSWER HA · 3

$|z-1|$ and $|z+1|$ are the distances from z to the points 1 and -1. Points equally far from both lie on the perpendicular bisector of the segment from -1 to 1: the imaginary axis. With $z = x + yi$: $(x-1)^2 + y^2 = (x+1)^2 + y^2$, so $-4x = 0$, $x = 0$: the imaginary axis again.

MARKING SCHEME

- ✓ The moduli are distances to 1 and -1.
- ✓ Equally far: the perpendicular bisector, the imaginary axis.
- ✓ The algebra gives $x = 0$.

8 Activity: Four quarter turns

(not marked)

WHAT STUDENTS SHOULD FIND

$3 + 2i$, $-2 + 3i$, $-3 - 2i$, $2 - 3i$, and back to $3 + 2i$: a square centered on O, each corner a quarter turn from the last; it closes because $i^4 = 1$.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	12	22	4	**43**

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