Unit 7 · Quadratic Equation

WORKSHEET

Full marks 41

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 41

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

coefficient	degree	discriminant	irrational	product	rational	roots	sum	symmetric	vertex

The numbers that satisfy a quadratic equation are its _____. The number $b^2 - 4ac$ is its _____. When a, b, c are rational and $b^2 - 4ac$ is a positive perfect square, the roots are _____; when it is positive but not a perfect square, they are _____. $-\frac{b}{a}$ is the _____ of the roots and $\frac{c}{a}$ their _____. An expression in α and β that is unchanged when they are swapped is _____. A polynomial equation of _____ n has exactly n roots, counting complex and repeated ones.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. If $b^2 - 4ac < 0$, the graph of $y = ax^2 + bx + c$ does not meet the X-axis. ☐

ii. If $b^2 - 4ac > 0$, the roots are always rational. ☐

iii. The roots of $ax^2 + bx + c = 0$ are reciprocals of each other when $b = 0$. ☐

iv. If $\frac{c}{a} < 0$, the roots are real and of opposite signs. ☐

v. $\alpha - \beta$ is a symmetric expression in the roots. ☐

vi. A cubic equation with real coefficients has at least one real root. ☐

3 Multiple choice [6]

Choose the correct option for each of the following questions.

i. The sum of the roots of $ax^2 + bx + c = 0$ is: [1]

a. $\frac{b}{a}$ **b.** $\frac{c}{a}$

c. $-\frac{b}{a}$ **d.** $-\frac{c}{a}$

ii. The quadratic equation whose roots are 3 and -5 is: [1]

a. $x^2 + 2x - 15 = 0$ **b.** $x^2 - 2x - 15 = 0$

c. $x^2 + 2x + 15 = 0$ **d.** $x^2 - 8x - 15 = 0$

iii. The equation $x^2 + 4x + 7 = 0$ has: **[1]**

a. two real roots
b. one repeated real root
c. two negative real roots
d. no real roots

iv. For what positive value of k does $x^2 - kx + 9 = 0$ have equal roots? **[1]**

a. 3
b. 6
c. 9
d. 36

v. If one root of $x^2 - 5x + k = 0$ is 2, the other root and k are: **[1]**

a. 3 and 6
b. 3 and 5
c. -3 and -6
d. 7 and 14

vi. $x^2 - 2mx + m + 6 = 0$ has real roots exactly when: **[1]**

a. $-2 \leq m \leq 3$
b. $m \geq 3$ only
c. $m \leq -2$ or $m \geq 3$
d. for every real m

4 Short answers **[12]**

Write short answers to the following questions.

a. Without solving, explain why $3x^2 + 5x - 7 = 0$ has one positive and one negative root. **[2]**

b. Diya finds, for the roots α, β of $2x^2 - 3x - 4 = 0$: **[2]**
"$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - \alpha\beta = \frac{9}{4} + 2 = \frac{17}{4}$."
Find her mistake and give the right value.

c. One root of $3x^2 + kx - 8 = 0$ is 2. Find k and the other root. **[2]**

d. If one root of $ax^2 + bx + c = 0$ is twice the other, prove that $2b^2 = 9ac$. Check it with $x^2 - 6x + 8 = 0$. **[3]**

e. Prove that the roots of $x^2 - 2(a + b)x + 2(a^2 + b^2) = 0$ are not real unless $a = b$, and find them when $a = b$. **[3]**

5 Structured question **[5]**

Answer all the parts.

a. Find $k \neq 0$ so that $x^2 - 5x + k = 0$ and $x^2 - 7x + 2k = 0$ have a common root. Find the common root. [3]

b. Find p and q so that $2x^2 + px + q = 0$ has both its roots common with $x^2 - 5x + 6 = 0$. [2]

6 Long question [8]

Write a long answer to the following question.

a. For what value of k does $(k-2)x^2 + 2(k-2)x + 2 = 0$ have equal roots? Find the roots for that k. [3]

b. If α, β are the roots of $3x^2 - 4x + 1 = 0$, form the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [3]

c. Sneha has 50 m of wire to fence a rectangular garden. Using the discriminant, decide whether its area can be 150 m^2, and whether it can be 160 m^2. [2]

7 Think [3]

Kushal says: "There is no quadratic equation with roots $1 + \sqrt{2}$ and 3." Kriti says: "There is: $(x - 1 - \sqrt{2})(x - 3) = 0$." Who is right? What extra condition makes Kushal right, and why?

Science Playground, Katunje, Bhaktapur
https://science-playground.com/learn/grade-11-math/quadratic-equation

8 Activity: Lifting the parabola

(not marked)

For each value of c, find the discriminant of $x^2 - 2x + c = 0$ and its roots, then sketch $y = x^2 - 2x + c$ on one pair of axes.

c	D	The roots	Where the graph meets the X-axis
-3			
0			
1			
2			
3			

What happens to the roots as c grows? At what value of c do they stop being real?