Unit 7 · Quadratic Equation

HARDER PRACTICE QUESTIONS

Full marks 26

Write your answers on separate paper. Show your working. Don't solve an equation where its sum and product of roots will do.

1 Where the roots lie [6]

The equation $x^2 - 2kx + 2k + 3 = 0$ depends on a real number k.

a. For which values of k are its roots real?

b. For which values of k are both roots positive? Both negative? Of opposite signs?

c. Rabina says: "At $k = -2$ the sum of the roots is negative, so both roots are negative." Is she right? Find the roots at $k = -2$.

2 Never real, except once [4]

a, b, c, d are real numbers, $a \neq 0$, $a^2 + b^2 \neq 0$.

a. Prove that $(a^2 + b^2)x^2 + 2(ac + bd)x + c^2 + d^2 = 0$ has no real roots unless $ad = bc$.

b. When $ad = bc$, show that its roots are equal, and that they are $-\frac{c}{a}$. Check with $a = 1$, $b = 2$, $c = 3$, $d = 6$.

3 New roots from old [6]

α and β are the roots of $2x^2 - 5x + 1 = 0$.

a. Without solving, form the equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, with whole-number coefficients.

b. Aisha finds the new product as $\alpha\beta + \frac{1}{\alpha\beta} = \frac{5}{2}$. What has she left out?

c. Is there an equation $x^2 + px + q = 0$ ($q \neq 0$) whose roots α, β give back the same two numbers, that is, $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$ are α and β in some order? Find every one, and check it.

4 After the common root [5]

$x^2 + ax + b = 0$ and $x^2 + bx + a = 0$, with $a \neq b$, have exactly one root in common.

a. Find the common root, and show that $a + b + 1 = 0$.

b. Find the root of each equation that is not shared, and the equation, with a the only letter in it, whose roots are these two.

c. Can these two roots both be positive? Both negative? Find the values of a for each, and check your equation of b. with $a = 2$.

5 A garden against a wall [5]

Nuwan has 30 m of fencing for a rectangular garden against the long wall of his house: the wall is the fourth side. Let each side at right angles to the wall be x m.

a. Write the area A in terms of x, and the equation for a given A.

b. Using the discriminant, decide whether the area can be 100 m^2, 112 m^2 and 120 m^2. Give the sides where it can.

c. Find the largest area possible, and its shape. Nuwan expects a square to be best, as it is for a garden fenced on all four sides. Is it?