Unit 7 · Quadratic Equation

ANSWER KEY · FOR TEACHERS

Full marks 41

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 4

1. roots
2. discriminant
3. rational
4. irrational
5. sum
6. product
7. symmetric
8. degree

Each carries 0.5 mark.

2 True or false

U · 3

i. ✓ **True**

ii. ✗ **False**: Only when $b^2 - 4ac$ is a perfect square (and a, b, c are rational); $x^2 - 4x + 1 = 0$ has $D = 12$ and roots $2 \pm \sqrt{3}$.

iii. ✗ **False**: They are reciprocals when $c = a$ (product 1); $b = 0$ makes them equal and opposite.

iv. ✓ **True**

v. ✗ **False**: Swapping gives $\beta - \alpha$, its negative; $(\alpha - \beta)^2$ is symmetric.

vi. ✓ **True**

Each carries 0.5 mark.

3 Multiple choice

i. **c.** $-\dfrac{b}{a}$ — K · 1

$a(x - \alpha)(x - \beta) = ax^2 - a(\alpha + \beta)x + a\alpha\beta$, so $-a(\alpha + \beta) = b$. $\dfrac{c}{a}$ is the product.

ii. **a.** $x^2 + 2x - 15 = 0$ — U · 1

Sum -2, product -15: $x^2 - (-2)x + (-15) = 0$. $x^2 - 2x - 15 = 0$ has roots -3 and 5: the sign of the sum turned round.

iii. **d.** no real roots — U · 1

$D = 16 - 28 = -12 < 0$. The sum -4 and product 7 would suit two negative roots only if they were real.

iv. **b.** 6 — A · 1

$k^2 - 36 = 0$, so $k = 6$, and the root is 3 twice. 36 is k^2; 3 is the root.

v. **a.** 3 and 6 — A · 1

The sum is 5, so the other root is 3, and k is the product, 6.

vi. **c.** $m \leq -2$ or $m \geq 3$ — HA · 1

$\dfrac{D}{4} = m^2 - m - 6 = (m - 3)(m + 2) \geq 0$, which holds outside -2 and 3. At $m = 0$, $x^2 + 6 = 0$ has no real roots.

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4 Short answers

a. MARKING SCHEME U · 2

✓ The roots are real ($D > 0$).

✓ The product $-\frac{7}{3} < 0$: opposite signs.

b. MARKING SCHEME U · 2

✓ $-2\alpha\beta$, not $-\alpha\beta$.

✓ $\frac{25}{4}$.

c. MARKING SCHEME A · 2

✓ $k = -2$.

✓ The other root $-\frac{4}{3}$.

d. MARKING SCHEME A · 3

✓ $3\alpha = -\frac{b}{a}$ and $2\alpha^2 = \frac{c}{a}$.

✓ α eliminated: $2b^2 = 9ac$.

✓ The check: $72 = 72$.

e. MARKING SCHEME A · 3

✓ $\frac{D}{4} = (a+b)^2 - 2(a^2+b^2)$.

✓ $= -(a-b)^2 < 0$ for $a \neq b$.

✓ Both roots $2a$ when $a = b$.

5 Structured question

ANSWER A · 5

a. Subtract the first from the second: $-2x + k = 0$, so the common root is $x = \frac{k}{2}$. In the first: $\frac{k^2}{4} - \frac{5k}{2} + k = 0$, so $\frac{k^2}{4} = \frac{3k}{2}$ and, as $k \neq 0$, $k = 6$. The common root is 3: $(x-2)(x-3)$ and $(x-3)(x-4)$. ✓

b. $\frac{2}{1} = \frac{p}{-5} = \frac{q}{6}$, so $p = -10$, $q = 12$.

MARKING SCHEME

✓ Subtracting: the common root $\frac{k}{2}$.

✓ $k = 6$.

✓ The common root 3.

✓ Both roots common: $\frac{2}{1} = \frac{p}{-5} = \frac{q}{6}$.

✓ $p = -10$, $q = 12$.

6 Long question

ANSWER

a. It is a quadratic only if $k \neq 2$. $\frac{D}{4} = (k-2)^2 - 2(k-2) = (k-2)(k-4) = 0$, so $k = 4$. Then $2x^2 + 4x + 2 = 2(x+1)^2 = 0$: both roots are -1.

b. $\alpha + \beta = \frac{4}{3}$, $\alpha\beta = \frac{1}{3}$, $\alpha^2 + \beta^2 = \frac{16}{9} - \frac{2}{3} = \frac{10}{9}$. New sum $\frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{10}{3}$, new product 1: $x^2 - \frac{10}{3}x + 1 = 0$, that is $3x^2 - 10x + 3 = 0$.

c. A side of x m leaves $25 - x$ m: $x(25-x) = A$, that is $x^2 - 25x + A = 0$, with $D = 625 - 4A$. For 150: $D = 25 > 0$, so yes (15 m by 10 m). For 160: $D = -15 < 0$: no rectangle.

MARKING SCHEME

✓ a. $k \neq 2$, and $\frac{D}{4} = (k-2)(k-4)$.

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✓ $k = 4$. A · 8

✓ Both roots -1.

✓ b. $\alpha + \beta$, $\alpha\beta$ and $\alpha^2 + \beta^2$.

✓ New sum $\frac{10}{3}$, new product 1.

✓ $3x^2 - 10x + 3 = 0$.

✓ c. $x^2 - 25x + A = 0$, $D = 625 - 4A$.

✓ 150 m^2 possible ($D = 25$), 160 m^2 not ($D = -15$).

7 Think

ANSWER HA · 3

Kriti's equation, $x^2 - (4 + \sqrt{2})x + 3 + 3\sqrt{2} = 0$, has those roots, so such an equation exists. But its coefficients are irrational. If the coefficients must be **rational**, Kushal is right: then irrational roots come in pairs $p \pm \sqrt{q}$ (the sum $-\frac{b}{a}$ and the product $\frac{c}{a}$ are rational only when the $\sqrt{2}$ cancels), so $1 + \sqrt{2}$ would bring $1 - \sqrt{2}$, and a quadratic has no room for a third root, 3.

MARKING SCHEME

✓ Kriti's equation works, with irrational coefficients.

✓ With rational coefficients, $1 + \sqrt{2}$ needs $1 - \sqrt{2}$ as the other root.

✓ So with rational coefficients no such quadratic exists: Kushal is right then.

8 Activity: Lifting the parabola

(not marked)

WHAT STUDENTS SHOULD FIND

$D = 16, 4, 0, -4, -8$; roots -1 and 3, 0 and 2, 1 twice, none real, none real. The same parabola lifted: the two crossings move together, meet at $x = 1$ when $c = 1$ ($D = 0$), and for $c > 1$ the graph misses the X-axis.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	9	23	4	**41**

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