Unit 6 · Matrices and Determinants

WORKSHEET

Full marks 43

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 43

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

adjoint	**cofactor**	**diagonal**	**identity**	**minor**	**singular**	**skew-symmetric**	**symmetric**
transpose	**triangular**						

The matrix whose rows are the columns of A is its _____. A square matrix with $A^{\mathrm{T}} = A$ is _____, and one with $A^{\mathrm{T}} = -A$ is _____. The determinant left when row i and column j are struck out is the _____ M_{ij}; with the sign $(-1)^{i+j}$ it is the _____ C_{ij}. The transpose of the matrix of cofactors is the _____. A matrix with $|A| = 0$ is _____ and has no inverse. The determinant of a _____ matrix is the product of its diagonal entries.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. Every entry on the leading diagonal of a skew-symmetric matrix is 0. ☐

ii. $(AB)^{\mathrm{T}} = A^{\mathrm{T}}B^{\mathrm{T}}$. ☐

iii. For a 3×3 matrix, $|2A| = 2|A|$. ☐

iv. Swapping two rows of a determinant changes its sign. ☐

v. $|A + B| = |A| + |B|$ for any square matrices of the same order. ☐

vi. A matrix with two proportional rows has no inverse. ☐

3 Multiple choice [6]

Choose the correct option for each of the following questions.

i. The adjoint adj A of a square matrix A is: [1]

a. the matrix of cofactors of A

b. the transpose of the matrix of cofactors of A

c. the transpose of A

d. the matrix of minors of A

ii. For $\begin{pmatrix} 3 & 1 & -2 \\ 0 & 4 & 5 \\ 2 & -1 & 6 \end{pmatrix}$, the cofactor C_{32} is: [1]

a. 15 **b.** 5

c. -15 **d.** -5

iii. A 3×3 determinant has the value D. Two of its rows are swapped, and then one row is multiplied by 3. The new value is: [1]

a. $-3D$ **b.** $3D$

c. $-27D$ **d.** $-D$

iv. $\begin{pmatrix} 1 & 2 & 3 \\ 2 & k & 6 \\ 0 & 1 & 4 \end{pmatrix}$ has no inverse when k is: [1]

a. 2 **b.** -4

c. 6 **d.** 4

v. A is a 3×3 matrix with $|A| = 3$. Then $|\text{adj } A|$ is: [1]

a. 3 **b.** 27

c. 9 **d.** $\frac{1}{3}$

vi. A is a 3×3 matrix of real numbers with $A^3 = 2I$. Then $|A|$ is: [1]

a. 2 **b.** 8

c. $\sqrt[3]{2}$ **d.** $\frac{1}{2}$

4 Short answers [14]

Write short answers to the following questions.

a. Explain why a matrix A with $|A| = 0$ can have no inverse. [2]

b. Prashant evaluates $|A|$ for $A = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 0 & -2 \\ 1 & 4 & 5 \end{pmatrix}$ down column 2: [2]

"$|A| = 4 \times (2 \times (-2) - 1 \times 3) = 4 \times (-7) = -28$."

Find his mistake and give the right value.

c. B is a 3×3 matrix with $|\text{adj } B| = 16$ and $|B| > 0$. Find $|B|$ and $|2B^{-1}|$. [2]

d. Find k if the point $(k, 1)$ lies on the line through $(-2, 3)$ and $(4, -1)$, using a determinant. **[2]**

e. Express $S = \begin{pmatrix} 3 & -2 & 4 \\ 6 & 1 & 0 \\ 2 & 5 & 7 \end{pmatrix}$ as the sum of a symmetric and a skew-symmetric matrix. **[3]**

f. Without expanding, show that the determinant of $\begin{pmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{pmatrix}$ is 0. Name the property used at each step. **[3]**

5 Structured question **[5]**

Answer all the parts.

$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

a. Show that $A^2 - 5A + 4I = O$. [3]

b. Hence find A^{-1}, without finding the adjoint. [2]

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6 Long question [8]

Write a long answer to the following question.

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}.$$

a. Find adj A. [3]

b. Find A^{-1} and check that $AA^{-1} = I$. [3]

c. Verify that $(A^{\mathrm{T}})^{-1} = (A^{-1})^{\mathrm{T}}$. [2]

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7 Think [3]

Kriti wants a key matrix A of whole numbers for a code in threes, whose inverse is also of whole numbers. Show that if $|A| = \pm 1$, then A^{-1} has whole-number entries; and that if both A and A^{-1} have whole-number entries, then $|A|$ must be 1 or -1.

8 Activity: What each change does (not marked)

Start with $A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 0 \\ 4 & 5 & 3 \end{pmatrix}$. Make each change to A, then find the new determinant by expanding (the easiest line first).

The matrix	Its determinant	Compared with the first row
A		
A^{T}		
A with rows 1 and 2 swapped		
A with $R_2 \to R_2 - 3R_1$		
A with row 1 doubled		
$2A$		

Which property of determinants does each row show?