Unit 5 · Sequence and Series

HARDER PRACTICE QUESTIONS

Full marks 28

Write your answers on separate paper. Show your working. A calculator may be used where a power or a logarithm has to be worked out.

1 A sum with one term too many [5]

The sum of the first n terms of a series is $S_n = 3n^2 + 2n + 1$.

a. Find t_1, t_2, t_3 and a formula for t_n. Sujana says: "$t_n = S_n - S_{n-1} = 6n - 1$, so it is an AP with first term 5." Is she right?

b. Show that if $S_n = pn^2 + qn + c$ for all $n \geq 1$, the series is an AP exactly when $c = 0$, and then give its first term and common difference in terms of p and q.

2 The three means at work [5]

a. For two positive numbers, $A - G = 2$ and $G - H = \dfrac{8}{5}$. Find A, G, H and the two numbers.

b. Simran wants to fence a rectangular kitchen garden of area 400 m^2. Use $A \geq G$ to find the least length of fence, and the shape that needs it.

c. With 60 m of fence instead, what is the largest area she can enclose?

3 Paying the loan off sooner [5]

A family borrows Rs 20,00,000 at 10% a year, interest $i = \dfrac{0.10}{12}$ a month, repaid at the end of each month. Repaying in 15 years costs Rs 21,492 a month.

a. Using the value today of the installments (a GP), show that with an installment of E for n months, $(1 + i)^{-n} = 1 - \dfrac{20{,}00{,}000\ i}{E}$.

b. The family pays Rs 25,000 a month instead. How many months does it take to repay? (Use logarithms; the last payment is smaller.)

c. Abinash says: "We pay Rs 3,508 more every month, so over the loan we pay $3{,}508 \times 180 \approx$ Rs 6.3 lakh more in all." Estimate the interest saved, and explain why he is wrong.

4 Three kinds in a row [4]

a, b, c are in AP, b, c, d are in GP, and c, d, e are in HP (all positive). Prove that a, c, e are in GP.

5 A ball that bounces for ever [5]

Lhakpa drops a ball from 10 m. Each bounce takes it back to $\dfrac{3}{4}$ of the height it fell from. A fall (or a rise) through height h takes $\sqrt{\dfrac{2h}{g}}$ seconds; take $g = 9.8$ m/s^2.

a. Find the total distance the ball travels.

b. Show that the times of successive falls form a GP, and give its ratio.

c. Find the total time before the ball comes to rest. Lhakpa uses the ratio $\dfrac{3}{4}$ for the times and gets 10 s: what has he missed?

6 For which x does it add up? [4]

$1 + \frac{x-1}{x+1} + \left(\frac{x-1}{x+1}\right)^2 + \cdots$ (with $x \neq -1$).

a. For which real x has this series a sum? Read the condition as a statement about distances on the number line (Unit 2).

b. Find the sum in terms of x, and the value of x for which it is 3.