Unit 4 · Curve Sketching

WORKSHEET

Full marks 42

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 42

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

amplitude	asymptote	decreasing	even	increasing	intercept	odd	origin	period	vertex

A function with $f(-x) = f(x)$ is _____; its graph is symmetric about the Y-axis. One with $f(-x) = -f(x)$ is _____; its graph is symmetric about the _____. The least positive T with $f(x + T) = f(x)$ for all x is the _____ of f. The greatest height of $a \sin bx$, $|a|$, is its _____. A line that a curve comes ever closer to but never reaches is an _____. The highest or lowest point of a parabola is its _____. A function with $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ is _____.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. $\cos x$ is an even function. ☐

ii. $x^3 + 1$ is an odd function. ☐

iii. The period of $5 \sin 3x$ is $\frac{2\pi}{5}$. ☐

iv. An odd function defined at $x = 0$ passes through the origin. ☐

v. $\frac{1}{x}$ is decreasing on its whole domain. ☐

vi. The curve $y^2 = 4x$ is symmetric about the Y-axis. ☐

3 Multiple choice [6]

Choose the correct option for each of the following questions.

i. A function f is odd if, for every x in its domain: [1]

a. $f(-x) = f(x)$ **b.** $f(-x) = -f(x)$

c. $f(x + 2\pi) = f(x)$ **d.** $f(x) \neq 0$

ii. Which function is even? [1]

a. $\frac{\sin x}{x}$ **b.** $x \cos x$

c. $\sin x + \cos x$ **d.** $x^3 + x$

iii. The curve $x^2 - xy + y^2 = 3$ is symmetric about: **[1]**

a. the X-axis
b. the Y-axis
c. the origin
d. none of the axes or the origin

iv. On which interval is $y = x^2 - 6x + 5$ decreasing? **[1]**

a. $(-\infty, 3]$
b. $[3, \infty)$
c. $(1, 5)$
d. $(-\infty, 1]$ and $[5, \infty)$

v. The vertex of $y = 2x^2 - 8x + 5$ is: **[1]**

a. $(4, 5)$
b. $(2, -3)$
c. $(-2, 29)$
d. $(2, 3)$

vi. The period of $\sin^4 x + \cos^4 x$ is: **[1]**

a. 2π
b. π
c. $\frac{\pi}{2}$
d. $\frac{\pi}{4}$

4 Short answers **[13]**

Write short answers to the following questions.

a. Test whether $g(x) = \frac{e^x - 1}{e^x + 1}$ is even, odd or neither. **[2]**

b. Test each curve for symmetry about the X-axis, the Y-axis and the origin: a. $xy = 9$; b. $x^4 + y^4 = 16$. **[2]**

c. Nima tests $f(x) = x^2 + \sin x$: "$f(-x) = x^2 - \sin x$, which is not $f(x)$, so f is odd." Find his mistake, and decide what f is, with a reason. **[3]**

d. Show from the definition that $f(x) = x^2 - 4x$ is decreasing on $(-\infty, 2]$. **[3]**

e. Find the period of $\tan 3x + \sin 2x$, and show that no smaller positive number is a period. **[3]**

5 Structured question **[5]**

Answer all the parts.

Sketch $y = 8 - 2x - x^2$, showing its vertex, axis of symmetry and intercepts, and state where it is increasing and where decreasing.

Answer:

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6 Long question [8]

Write a long answer to the following question.

$f(x) = x^3 - 3x^2$.

a. Is f even, odd or neither? [1]

b. Find its intercepts, and the sign of $f(x)$ on each interval they make. [3]

c. Describe the curve as $x \to \pm\infty$, find $f(-1)$, $f(1)$, $f(2)$ and $f(4)$, and sketch it. [3]

d. From your sketch, how many real roots has $x^3 - 3x^2 = -1$? [1]

Answer: _____

7 Think [3]

The day length in Kathmandu is about $L = 12.14 + 1.76 \sin\left(\frac{2\pi(t-80)}{365}\right)$ hours on day t of the year. A town at the same latitude **south** of the equator has its longest day when Kathmandu has its shortest. Write a formula for that town's day length, and say what changes in the graph and what doesn't.

8 Activity: Stretch it up, squeeze it sideways (not marked)

Fill in the table, then on squared paper plot both curves for $0 \le x \le 2\pi$ on the same axes.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$2 \sin x$									
$\sin 2x$									

What are the amplitude and the period of each?

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