Unit 4 · Curve Sketching

ANSWER KEY · FOR TEACHERS

Full marks 42

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 4

1. even
2. odd
3. origin
4. period
5. amplitude
6. asymptote
7. vertex
8. increasing

Each carries 0.5 mark.

2 True or false

U · 3

i. ✓ **True**

ii. ✗ **False**: It is neither: $f(1) = 2$ and $f(-1) = 0$, neither equal nor opposite.

iii. ✗ **False**: It is $\frac{2\pi}{3}$; the 5 changes only the amplitude.

iv. ✓ **True**

v. ✗ **False**: Only on $(-\infty, 0)$ and on $(0, \infty)$ separately: $-1 < 1$ and $\frac{1}{-1} < \frac{1}{1}$.

vi. ✗ **False**: About the X-axis: putting $-y$ for y leaves it unchanged; putting $-x$ for x doesn't.

Each carries 0.5 mark.

3 Multiple choice

i. **b.** $f(-x) = -f(x)$ — K · 1

The graph of an odd function is symmetric about the origin. The first is the test for an even function, the third for periodicity.

ii. **a.** $\frac{\sin x}{x}$ — U · 1

$\frac{\sin(-x)}{-x} = \frac{-\sin x}{-x} = \frac{\sin x}{x}$: odd divided by odd is even, as odd × odd is. $x \cos x$ and $x^3 + x$ are odd; $\sin x + \cos x$ is neither.

iii. **c.** the origin — U · 1

Putting $-x$ and $-y$ together, $(-x)(-y) = xy$, so the equation is unchanged: the origin. $y \to -y$ alone gives $x^2 + xy + y^2 = 3$, a different curve, and so does $x \to -x$: neither axis. (Swapping x and y leaves it unchanged too, so it is also symmetric about $y = x$.)

iv. **a.** $(-\infty, 3]$ — U · 1

It opens upward with its vertex at $x = -\frac{-6}{2} = 3$, so it falls to 3 and rises after. (1, 5) is where it is below the X-axis, a question of sign, not of direction.

v. **b.** $(2, -3)$ — A · 1

$x = -\frac{-8}{4} = 2$, and $y = 8 - 16 + 5 = -3$. $(-2, 29)$ takes $x = \frac{b}{2a}$ with the wrong sign; (4, 5) forgets the 2 in $2a$.

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vi. **c.** $\frac{\pi}{2}$ HA · 1

$\sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x = 1 - \frac{1}{2}\sin^2 2x = \frac{3}{4} + \frac{1}{4}\cos 4x$, whose period is $\frac{2\pi}{4} = \frac{\pi}{2}$. Each term alone has period π, but the sum repeats sooner. $\frac{\pi}{4}$ is half a period of $\cos 4x$.

4 Short answers

a. MARKING SCHEME U · 2

✓ $g(-x)$ simplified to $\frac{1 - e^x}{1 + e^x}$.

✓ $= -g(x)$: odd.

b. MARKING SCHEME U · 2

✓ a. The origin only.

✓ b. Both axes and the origin.

c. MARKING SCHEME U · 3

✓ "Not even" doesn't mean odd: odd needs $f(-x) = -f(x)$.

✓ $f(-x)$ compared with $-f(x)$ (or a point that fails both).

✓ Neither.

d. MARKING SCHEME A · 3

✓ $x_1 < x_2 \leq 2$ taken, $f(x_2) - f(x_1)$ factorized.

✓ $x_2 - x_1 > 0$ and $x_1 + x_2 - 4 < 0$.

✓ So $f(x_2) < f(x_1)$: decreasing.

e. MARKING SCHEME A · 3

✓ The periods $\frac{\pi}{3}$ and π, so π is a period of the sum.

✓ T must be a multiple of $\frac{\pi}{3}$ (where tan is undefined).

✓ Then $\sin 2x$ needs a multiple of π: the period is π.

5 Structured question

ANSWER A · 5

$a = -1 < 0$: it opens downward. Vertex at $x = -\frac{-2}{2(-1)} = -1$, $y = 8 + 2 - 1 = 9$: **(−1, 9)**, the highest point; axis $x = -1$. Y-intercept (0, 8). X-intercepts: $x^2 + 2x - 8 = (x + 4)(x - 2) = 0$: **(−4, 0) and (2, 0)**, 3 either side of the axis. The sketch: a parabola opening down through these points. **Increasing on $(-\infty, -1]$, decreasing on $[-1, \infty)$.**

MARKING SCHEME

✓ Opens downward; vertex $(-1, 9)$.

✓ Axis $x = -1$.

✓ Intercepts $(0, 8)$, $(-4, 0)$, $(2, 0)$.

✓ A sketch with these, symmetric about the axis.

✓ Increasing on $(-\infty, -1]$, decreasing on $[-1, \infty)$.

6 Long question

ANSWER

a. $f(1) = -2$ and $f(-1) = -4$: neither equal nor opposite, so **neither**.

b. $f(x) = x^2(x - 3)$: Y-intercept (0, 0); X-intercepts at $x = 0$ (a double root: the curve touches) and $x = 3$. $x^2 \geq 0$, so the sign is that of $x - 3$: **negative on $(-\infty, 0)$ and $(0, 3)$, positive on $(3, \infty)$.**

c. $a = 1 > 0$: down on the left, up on the right. $f(-1) = -4$, $f(1) = -2$, $f(2) = -4$, $f(4) = 16$. It rises to touch the axis at the origin, dips below between 0 and 3 (lowest near $x = 2$), and climbs through (3, 0).

d. The line $y = -1$ cuts the curve **three times**: between −1 and 0, between 0 and 2, and between 2 and 3 (the roots are about −0.53, 0.65 and 2.88).

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MARKING SCHEME A · 8

- ✓ a. Neither, shown with a point.
- ✓ b. $x^2(x - 3)$; intercepts (0, 0) and (3, 0).
- ✓ The curve touches at 0 (double root).
- ✓ The signs: negative left of 3 (except at 0), positive right of 3.
- ✓ c. The ends: down on the left, up on the right.
- ✓ The four values.
- ✓ A sketch through them, touching at 0 and crossing at 3.
- ✓ d. Three real roots.

7 Think

ANSWER HA · 3

Its days are Kathmandu's half a year later: $L = 12.14 - 1.76 \sin\left(\frac{2\pi(t - 80)}{365}\right)$ (the same as shifting t by $\frac{365}{2}$, since $\sin(\theta + \pi) = -\sin\theta$). The wave is turned upside down about its middle line: the middle 12.14 h, the amplitude 1.76 h and the period 365 days stay; the longest day moves to about day 354 and the shortest to about day 171.

MARKING SCHEME

- ✓ The formula with -1.76 (or a shift of half a period).
- ✓ Middle, amplitude and period unchanged.
- ✓ Longest and shortest days swapped (about day 354 and day 171).

8 Activity: Stretch it up, squeeze it sideways

(not marked)

WHAT STUDENTS SHOULD FIND

$2 \sin x$: 0, 1.41, 2, 1.41, 0, −1.41, −2, −1.41, 0: amplitude 2, period 2π. $\sin 2x$: 0, 1, 0, −1, 0, 1, 0, −1, 0: amplitude 1, period π, two waves in $[0, 2\pi]$. The 2 outside stretches the wave up; the 2 inside squeezes it sideways.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	13	20	4	**42**

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