Unit 3 · Function

WORKSHEET

Full marks 41

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 41

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

algebraic	**codomain**	**composite**	**inverse**	**natural domain**	**one-to-one**	**onto**	**periodic**	**range**	**transcendental**

The largest set of real numbers for which a formula gives a real number is its _____. The set of images that really occur is the _____; the set the images are allowed to lie in is the _____. A function whose different inputs always have different images is _____, and one whose range is the whole codomain is _____. Only a function that is both has an _____. $g(f(x))$ is the _____ $g \circ f$. Exponential, logarithmic and trigonometric functions are _____.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. The domain of $\ln x$ is $(0, \infty)$. ☐

ii. $f^{-1}(x) = \dfrac{1}{f(x)}$. ☐

iii. In $(g \circ f)(x)$, g is applied first. ☐

iv. $f(x) = x^2, x \in \mathbb{R}$, has the inverse $\sqrt{x}$. ☐

v. The graph of f^{-1} is the mirror image of the graph of f in the line $y = x$. ☐

vi. The range of 2^x is $[0, \infty)$. ☐

3 Multiple choice [6]

Choose the correct option for each of the following questions.

i. The domain of $f(x) = \dfrac{1}{x-4}$ is: [1]

a. $\mathbb{R}$ **b.** $\mathbb{R} - \{4\}$

c. $\mathbb{R} - \{0\}$ **d.** $(4, \infty)$

ii. The inverse of $f(x) = 2^x$ is: [1]

a. 2^{-x} **b.** $\log_x 2$

c. $\log_2 x$ **d.** x^2

iii. The graph of $y = \log_3(x - 2)$ crosses the x-axis at: **[1]**

a. $x = 1$ **b.** $x = 2$

c. $x = 3$ **d.** nowhere

iv. If $f(x) = \dfrac{3x - 2}{x + 1}$, $x \neq -1$, then $f^{-1}(x)$ is: **[1]**

a. $\dfrac{x + 1}{3x - 2}$ **b.** $\dfrac{x + 2}{3 - x}$

c. $\dfrac{x - 2}{3 + x}$ **d.** $\dfrac{3x + 2}{x - 1}$

v. The range of $f(x) = \dfrac{x^2}{x^2 + 1}$ is: **[1]**

a. $[0, 1]$ **b.** $[0, 1)$

c. $(0, 1)$ **d.** $[0, \infty)$

vi. $f(x) = ax + b$ and $g(x) = cx + d$. Then $f \circ g = g \circ f$ exactly when: **[1]**

a. $ad = bc$ **b.** $a = c$ and $b = d$

c. $b(c - 1) = d(a - 1)$ **d.** $a + b = c + d$

4 Short answers **[12]**

Write short answers to the following questions.

a. Find the domain and range of $g(x) = \sqrt{9 - x^2}$. **[2]**

b. Find the domain of $f(x) = \ln(x^2 - 5x + 6)$. **[2]**

c. $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$. Saugat writes: "$(f \circ g)(x) = (\sqrt{x})^2 - 1 = x - 1$, so $(f \circ g)(-4) = -5$." What is wrong? Give the domains of $f \circ g$ and $g \circ f$. **[3]**

d. $f(x) = 2x + 3$ and $(g \circ f)(x) = 4x^2 + 12x + 10$. Find $g(x)$. **[2]**

e. Find the inverse of $g(x) = \log_3(2x + 1)$, and state the domain and range of g^{-1}. **[3]**

5 Structured question **[5]**

Answer all the parts.

$f(x) = 2x - 3$ and $g(x) = x^2 + 2$.

a. Find $f \circ g$ and $g \circ f$, and show that they are different. [2]

b. Find the values of x for which $(f \circ g)(x) = (g \circ f)(x)$, and check one. [3]

6 Long question **[8]**

Write a long answer to the following question.

$f(x) = \dfrac{x+1}{x-2}$ and $g(x) = 2x - 1$.

a. Find the domain and range of f. [2]

b. Find f^{-1} and g^{-1}. [2]

c. Find $(g \circ f)^{-1}$ in two ways, directly and as $f^{-1} \circ g^{-1}$, and show that they agree. [4]

Science Playground, Katunje, Bhaktapur
https://science-playground.com/learn/grade-11-math/function

7 Think [3]

An earthquake of magnitude M releases about $E = 10^{1.5M + 4.8}$ joules.

a. Show that raising M by 2 multiplies E by 1000.

b. One earthquake releases twice the energy of another. By how much is its magnitude larger? ($\log_{10} 2 \approx 0.301$)

8 Activity: A function and its mirror (not marked)

Fill in the table, then on squared paper plot the points of $y = 2^x$ and of its inverse $y = \log_2 x$, with the line $y = x$.

x	-2	-1	0	1	2	3
2^x						
point on $y = \log_2 x$						

Where does each graph cross an axis? What are the domain and range of each?