Unit 3 · Function

HARDER PRACTICE QUESTIONS

Full marks 27

Write your answers on separate paper. Show your working. Write domains and ranges as intervals or as $\mathbb{R} - \{\ldots\}$.

1 Ranges that hide a gap [5]

a. Prapti says: "$f(x) = \dfrac{x^2 - 1}{x - 1}$ is just $x + 1$, so its range is $\mathbb{R}$." Find the domain and the range of f, and say where she goes wrong.

b. Find the range of $g(x) = \dfrac{x + 1}{x^2 + 3}$. For which x are its largest and smallest values reached?

2 Machines in a row [4]

$f(x) = \sqrt{x}$, $g(x) = \ln x$ and $h(x) = 1 - x$.

a. Write $(f \circ g \circ h)(x)$ and find its domain and range.

b. Write $(g \circ f \circ h)(x)$ and find its domain and range.

c. Dipesh says the domain in a. is $(-\infty, 1)$, "because the log needs $1 - x > 0$". What has he missed?

3 Half a parabola and its mirror [5]

$f(x) = x^2 - 2x$ on the domain $[1, \infty)$.

a. Show that f is one-to-one on this domain, and find its range.

b. Find $f^{-1}(x)$ with its domain. Rojina gets two formulas, $1 \pm \sqrt{x + 1}$. Which one is right, and why?

c. Solve $f(x) = f^{-1}(x)$.

d. Show that if an increasing function f and its inverse meet at (a, b), then $a = b$.

4 A function that undoes itself [4]

a. For which value of k is $f(x) = \dfrac{3x + 5}{x + k}$ its own inverse? Prove that your k works.

b. What does the graph of such a function look like beside the line $y = x$?

5 Twice the same machine [4]

Find every linear function $g(x) = ax + b$ with $(g \circ g)(x) = 4x + 3$ for all x. Check each. Tenzing finds one answer and stops: why should he have kept going?

6 Energies don't add like magnitudes [5]

An earthquake of magnitude M releases about $E = 10^{1.5M + 4.8}$ joules. A calculator may be used for the powers and logarithms.

a. Which releases more energy: ten earthquakes of magnitude 6.0, or one of magnitude 7.0? By what factor?

b. About how many earthquakes of magnitude 5.0 release as much energy as one of magnitude 7.8?

c. The two big shocks of 2015, 7.8 and 7.3, together released how much energy? What single magnitude would release that much? Why is it not 7.8 + 7.3, and why is it so little above 7.8?