Unit 2 · Real Numbers

WORKSHEET

Full marks 42

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 42

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

absolute value	**bounded**	**closed**	**irrational**	**neighborhood**	**open**	**rational**	**real line**
triangle inequality	**unbounded**						

A number such as $\sqrt{2}$, whose decimal neither ends nor repeats, is _____. Each real number is one point of the _____. $[a, b]$ is a _____ interval and (a, b) an _____ one; (a, ∞) is _____. The distance of x from 0 is its _____, $|x|$. The interval $(c - r, c + r)$, the numbers within r of c, is a _____ of c. $|x + y| \leq |x| + |y|$ is the _____.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. Between any two rational numbers there are irrational numbers. ☐

ii. $[2, \infty]$ is the set of real numbers $x \geq 2$. ☐

iii. $\sqrt{x^2} = x$ for every real x. ☐

iv. $|x| < -2$ for every real x. ☐

v. $|a - b| = |b - a|$ for all real a, b. ☐

vi. $|a + b| = |a| + |b|$ for all real a, b. ☐

3 Multiple choice [6]

Choose the correct option for each of the following questions.

i. The interval $(-2, 5]$ contains: [1]

a. -2 and 5
b. -2 but not 5
c. 5 but not -2
d. neither -2 nor 5

ii. $\sqrt{x^2}$ is equal to: [1]

a. x
b. $-x$
c. $|x|$
d. $\pm x$

iii. Which inequality describes $[-1, 5]$? [1]

a. $|x - 2| < 3$
b. $|x - 2| \leq 3$
c. $|x + 2| \leq 3$
d. $|x - 3| \leq 2$

iv. The number of integers x with $|x - 2| < 3$ is: [1]

a. 7
b. 6
c. 5
d. 3

v. The solution of $|x+3| = |x-5|$ is: **[1]**

a. $x = 4$ **b.** $x = 1$

c. $x = -1$ **d.** no real number

vi. For how many real x is $|x| + |x-4| = 4$? **[1]**

a. two **b.** none

c. infinitely many **d.** one

4 Short answers [13]

Write short answers to the following questions.

a. Write each set with an absolute value, as $|x-c| \leq r$ or $|x-c| > r$: **[2]**

a. $[-3, 7]$

b. $(-\infty, 1) \cup (5, \infty)$

b. Solve $|3x+1| = 8$. **[2]**

c. Anisha solved $|3-x| < 2$ like this: **[3]**

"$-2 < 3-x < 2$, so $-5 < -x < -1$, so $-5 < x < -1$."

Find her mistake, and write the right answer as an interval. Check it with one number.

d. Solve $|x-2| = 3x+6$. **[3]**

e. Prove that $|x - y| \geq |x| - |y|$ for all real x and y. **[3]**

5 Structured question **[5]**

Answer all the parts.

$A = [-3, 2)$, $B = (0, 5]$ and $C = (-\infty, 1]$.

a. Show A and B on one number line. [1]

b. Find $A \cup B$, $A \cap B$ and $B - A$. [2]

c. Find $A \cap B \cap C$ and $\overline{C}$ (in the real numbers). [2]

6 Long question **[8]**

Write a long answer to the following question.

a. Solve $|x - 1| + |x - 3| \leq 4$ by splitting the line at 1 and 3, and write the answer as an interval. [4]

b. Explain the answer with distances on the number line. [2]

c. Without working through the cases again, write the solution sets of $|x - 1| + |x - 3| \leq 2$ and $|x - 1| + |x - 3| \leq 1$. Give a reason. [2]

7 Think [3]

Aayusha says: "If $|x-3|<0.1$ and $|y-5|<0.1$, then $|(x+y)-8|<0.2$." Is she right? Prove it, or give numbers that show she is wrong. Can $|(x+y)-8|$ be as large as 0.19?

8 Activity: Center and radius (not marked)

Fill in each row. Then draw each interval on a number line, a filled dot for an end that is in and an open dot for one that is out.

Rows a to c are $|x-4|\le 3$, $|x+1|<2$ and $|2x-6|\le 4$; in row d, write $(-2, 8)$ as an inequality.

	Interval	Center	Radius	Ends in or out?
a.				
b.				
c.				
d.	$(-2, 8)$			

How do you read the center and the radius straight from an interval?