Unit 1 · Logic and Set

WORKSHEET

Full marks 41

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 41

1 Key words [4]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

contradiction	contrapositive	converse	De Morgan's	equivalent	hypothesis	inverse
open sentence	statement	tautology				

A sentence that is either true or false, but not both, is a _____. In $p \to q$, p is the _____ and q the conclusion. $\sim q \to \sim p$ is the _____ of $p \to q$, and $q \to p$ its _____. A compound statement that is true in every row of its truth table is a _____; one false in every row is a _____. Two statements with the same truth value in every row are logically _____. The law $\overline{A \cup B} = \overline{A} \cap \overline{B}$ is _____ law.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. "$2 + 3 = 6$" is a statement. ☐

ii. "$x^2 = 4$" is a statement. ☐

iii. The negation of "Sabina passed English and Nepali" is "Sabina failed English and failed Nepali". ☐

iv. $p \to q$ is true whenever p is false. ☐

v. A conditional and its converse always have the same truth value. ☐

vi. $A - B = A \cap \overline{B}$. ☐

3 Multiple choice [6]

Choose the correct option for each of the following questions.

i. $p \vee q$ is false when: [1]

a. p is true and q is false
b. p is false and q is true
c. both p and q are false
d. both p and q are true

ii. The converse of $\sim p \to q$ is: [1]

a. $p \to q$
b. $q \to \sim p$
c. $\sim q \to p$
d. $q \to p$

iii. $\sim(p \to q)$ is equivalent to: [1]

a. $\sim p \to \sim q$
b. $\sim p \vee q$
c. $p \wedge \sim q$
d. $\sim p \wedge q$

iv. The truth table of $(p \wedge \sim q) \vee (r \to s)$ has [1]

a. 4 rows **b.** 8 rows

c. 16 rows **d.** 32 rows

v. $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5\}$. Then $\overline{A \cup B}$ is: [1]

a. $\{1, 4, 5, 6\}$ **b.** $\{6\}$

c. $\{2, 3\}$ **d.** $\{1, 6\}$

vi. For sets A and B, $A \cup B = A \cap B$ exactly when: [1]

a. $A \subseteq B$ **b.** $A \cap B = \emptyset$

c. $A = B$ **d.** $B = U$

4 Short answers [12]

Write short answers to the following questions.

a. Write the negation of "If a number is divisible by 6, then it is even" in words, without using "it is not true that". [2]

b. p: "$\sqrt{9} = 3$" and q: "4 is a prime number". Find the truth values of $p \to q$ and $\sim q \leftrightarrow p$. [2]

c. Using a truth table, show that $p \to q \equiv \sim q \to \sim p$. What does this mean for a proof? [3]

d. $n(A) = 45$, $n(B) = 30$ and $n(A \cap B) = 12$. Find $n(A \cup B)$ and $n(B - A)$. [2]

e. Kushal wrote this proof that $\overline{A \cap B} = \overline{A} \cup \overline{B}$: **[3]**
"Take $U = \{1, 2, 3, 4\}$, $A = \{1, 2\}$, $B = \{2, 3\}$. Then $\overline{A \cap B} = \{1, 3, 4\}$ and $\overline{A} \cup \overline{B} = \{3, 4\} \cup \{1, 4\} = \{1, 3, 4\}$. They are equal, so the law is proved."
What is wrong with his proof? Is his working right?

5 Structured question **[5]**

Answer all the parts.

a. Construct the truth table of $(p \vee q) \rightarrow (p \wedge q)$. Is it a tautology, a contradiction or a contingency? [3]
b. In words, when is $(p \vee q) \rightarrow (p \wedge q)$ true? Which simpler statement is it equivalent to? [2]

6 Long question **[8]**

Write a long answer to the following question.

a. For any sets A, B and C, prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, giving the reason for each step. [4]
b. $U = \{1, 2, \ldots, 10\}$, $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$ and $C = \{4, 5, 6, 7\}$. Verify the law of a. with these sets. [2]
c. Using the laws of logic, not a truth table, show that $p \wedge (\sim p \vee q) \equiv p \wedge q$, naming each law. [2]

Science Playground, Katunje, Bhaktapur
https://science-playground.com/learn/grade-11-math/logic-and-set

7 Think [3]

A staircase light has two switches. p: "the bottom switch is up", q: "the top switch is up". Kriti's light is on exactly when $p \leftrightarrow q$ is true. Show that flipping either switch always changes the light, whatever the other switch is doing. Would a light on when $p \wedge q$ is true work as a staircase light?

8 Activity: Columns that agree (not marked)

Work out each compound in every row of the table, a column at a time. Then look for columns that are the same all the way down.

p	q	$\sim p \vee q$	$p \rightarrow q$	$\sim(p \wedge \sim q)$	$q \rightarrow p$
T	T				
T	F				
F	T				
F	F				

Which columns are the same? What does that tell you?