Unit 1 · Logic and Set

HARDER PRACTICE QUESTIONS

Full marks 25

Write your answers on separate paper. Show your working. Give a reason for every step of a proof.

1 When the road is wet [4]

Pratiksha argues: "If it rained last night, the road is wet. The road is wet. So it rained last night."

a. Let p: "it rained last night" and q: "the road is wet". Write her argument as one compound statement of the form (premises) $\rightarrow$ (conclusion).

b. Is that statement a tautology? Use a truth table, and point to the row that decides.

c. Keeping her conclusion, change one premise so that the argument becomes valid without simply assuming that conclusion, and say which law of reasoning in the unit it then is.

2 Splitting the hypothesis [4]

A shop's rule is: "If a customer pays in cash and buys more than Rs 1000 of goods, the customer gets a discount": $(p \wedge q) \rightarrow r$.

a. Using the laws of logic (no truth table), show that $(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \vee (q \rightarrow r)$, naming the law at each step.

b. Nischal says the rule means the same as $(p \rightarrow r) \wedge (q \rightarrow r)$. Give one row of truth values where the two differ, and say in words what his version would wrongly demand of the shop.

3 A vote of three [5]

Three members of a school committee, Aaryan, Bhumika and Dawa, vote yes or no. p, q, r: "Aaryan (Bhumika, Dawa) votes yes". A motion passes when **at least two** vote yes.

a. Using only $\wedge$ and $\vee$, write a compound statement $M(p, q, r)$ that is true exactly when the motion passes. Check it for the votes yes, no, yes.

b. The motion fails exactly when at least two vote no. Use this to explain why $\sim M(p, q, r) \equiv M(\sim p, \sim q, \sim r)$.

c. Write $\sim M(p, q, r)$ a second way, from your answer to a. by De Morgan's laws, and show that it says "at most one votes yes".

4 Is the difference associative? [4]

Shreya claims that $(A - B) - C = A - (B - C)$ for all sets A, B, C.

a. Show that she is wrong, with sets of your own listed in full.

b. One of the two sides is always a subset of the other. Which? Prove it, element by element.

5 A promise kept by most [4]

In a class of 60, 30 students play football (F) and 35 play cricket (C). For a student x, the statement "if x plays football, then x plays cricket" is false for exactly 12 students.

a. Which region of the Venn diagram are those 12 students? How many students play neither game?

b. For how many students is the statement true? Anil says 18, "the ones who play both". Explain his mistake.

6 By the contrapositive [4]

For integers m and n, prove that if $m + n$ is odd, then m is even or n is even.

a. Write the contrapositive, negating its parts correctly, and prove it.

b. Write the converse. Is it true? Prove it or give a counterexample.