Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 When the road is wet

ANSWER HA · 4

Tests: the conditional against its converse; valid reasoning as a tautology.

a. $[(p \to q) \wedge q] \to p$.

b.

p	q	$p \to q$	$(p \to q) \wedge q$	**whole**
T	T	T	T	T
T	F	F	F	T
F	T	T	T	**F**
F	F	T	F	T

Not a tautology: in row 3 (no rain, road wet: a water tanker passed) both premises are true and the conclusion false. Her argument treats $p \to q$ as if it were its converse $q \to p$.

c. Replace the first premise by its converse, "if the road is wet, it rained last night": $[(q \to p) \wedge q] \to p$, which is modus ponens, a tautology (Example 4 of the unit). (Replacing "the road is wet" by "it rained last night" gives $[(p \to q) \wedge p] \to p$, valid but only because the conclusion is already a premise: that is what "without simply assuming it" rules out.)

MARKING SCHEME

✓ a. $[(p \to q) \wedge q] \to p$.
✓ b. The truth table with the last column T, T, F, T.
✓ Not a tautology: the row p F, q T has true premises and a false conclusion (the converse used).
✓ c. The first premise changed to $q \to p$, and named: modus ponens.

2 Splitting the hypothesis

ANSWER A · 4

Tests: the conditional as $\sim p \vee q$; De Morgan inside a conditional; the tempting $\wedge$.

a. $(p \wedge q) \to r \equiv \sim(p \wedge q) \vee r$ (the conditional as an "or", Example 3)
$\equiv (\sim p \vee \sim q) \vee r$ (De Morgan)
$\equiv (\sim p \vee r) \vee (\sim q \vee r)$ (idempotent $r \vee r \equiv r$, then commutative and associative)
$\equiv (p \to r) \vee (q \to r)$ (the conditional as an "or", twice). ✓

b. p T, q F, r F (a cash customer buying less than Rs 1000 gets no discount): the rule $(p \wedge q) \to r$ is true (F $\to$ F), but $(p \to r) \wedge (q \to r)$ is false, since $p \to r$ is T $\to$ F. Nischal's version would demand a discount for **every** cash customer, and for every big purchase, not only for both together. (p F, q T, r F is the other such row.)

MARKING SCHEME

✓ $\sim(p \wedge q) \vee r$, then De Morgan.
✓ r written twice (idempotent), regrouped into $(\sim p \vee r) \vee (\sim q \vee r)$, each back to a conditional.
✓ A row where they differ, e.g. p T, q F, r F.
✓ His version gives a discount to every cash customer (or every big purchase).

3 A vote of three

ANSWER

Tests: building a statement from a rule; De Morgan applied to a long compound; reading what a negation means.

a. $M = (p \wedge q) \vee (q \wedge r) \vee (p \wedge r)$: some pair both vote yes. Yes, no, yes: $p \wedge r$ is T, so M is **T**: it passes, as two voted yes.

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b. Each member votes either yes or no, so among three votes, "fewer than two yes" is the same as "at least two no". "At least two no" is M with each vote replaced by its negation: $M(\sim p, \sim q, \sim r)$. So the negation of "passes" is $M(\sim p, \sim q, \sim r)$. (A check of all 8 rows agrees.) HA · 5

c. $\sim M \equiv \sim(p \wedge q) \wedge \sim(q \wedge r) \wedge \sim(p \wedge r)$ (De Morgan on the "or"s)
$\equiv (\sim p \vee \sim q) \wedge (\sim q \vee \sim r) \wedge (\sim p \vee \sim r)$ (De Morgan on each "and"). Each bracket says "not both of this pair voted yes": no pair voted yes together, so at most one member voted yes.

MARKING SCHEME

- ✓ a. $(p \wedge q) \vee (q \wedge r) \vee (p \wedge r)$.
- ✓ The check: yes, no, yes gives T (from $p \wedge r$).
- ✓ b. Fewer than two yes means at least two no, which is $M(\sim p, \sim q, \sim r)$.
- ✓ c. De Morgan twice: $(\sim p \vee \sim q) \wedge (\sim q \vee \sim r) \wedge (\sim p \vee \sim r)$.
- ✓ Read as "no pair both voted yes", i.e. at most one yes.

4 Is the difference associative?

ANSWER HA · 4

Tests: an example can disprove an identity; proving one inclusion from "and", "not".

a. For example $A = \{1, 2, 3\}$, $B = \{2\}$, $C = \{1, 2\}$: $(A - B) - C = \{1, 3\} - \{1, 2\} = \{3\}$, but $B - C = \emptyset$, so $A - (B - C) = \{1, 2, 3\}$. Not equal.

b. $(A - B) - C \subseteq A - (B - C)$. Take $x \in (A - B) - C$: then $x \in A$, $x \notin B$ and $x \notin C$ (difference, twice). Since $x \notin B$, x is not in $B - C$ (whose elements are all in B). So $x \in A$ and $x \notin B - C$: $x \in A - (B - C)$. ✓ (The other way fails: an element of $A \cap C$ is in the right side but not the left, as 1 is above.)

MARKING SCHEME

- ✓ a. Sets for which the two sides differ, both sides worked out.
- ✓ b. The left side is the subset.
- ✓ $x \in (A - B) - C$ gives $x \in A$, $x \notin B$, $x \notin C$.
- ✓ $x \notin B$ so $x \notin B - C$; hence $x \in A - (B - C)$.

5 A promise kept by most

ANSWER A · 4

Tests: sets speak logic ($\rightarrow$ fails only on $F - C$); the conditional true when the hypothesis is false.

a. $x \in F \rightarrow x \in C$ is false only when $x \in F$ and $x \notin C$: the region $F - C$, so $n(F - C) = 12$. Then $n(F \cap C) = 30 - 12 = 18$, $n(F \cup C) = 30 + 35 - 18 = 47$, and neither: $60 - 47 = \mathbf{13}$.

b. **48** $(60 - 12)$. The statement is true for everyone not in $F - C$: the 18 who play both, but also the 17 who play only cricket and the 13 who play neither, for whom the hypothesis is false, so the promise isn't broken. Anil counted only the rows where both parts are true.

MARKING SCHEME

- ✓ a. The 12 are $F - C$ (football and not cricket).
- ✓ Both 18, union 47, neither 13.
- ✓ b. 48 students.
- ✓ A conditional with a false hypothesis is true: the 17 cricket-only and 13 neither count too.

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6 By the contrapositive

ANSWER A · 4

Tests: proof by contrapositive; negating an "or" (De Morgan) inside it; a converse needs its own check.

a. The conclusion "m even or n even" negates, by De Morgan, to "m odd **and** n odd" (not "m odd or n odd"). Contrapositive: "if m and n are both odd, then $m + n$ is even". If $m = 2j + 1$ and $n = 2k + 1$, then $m + n = 2(j + k + 1)$, even. ✓ Since $p \to q \equiv \sim q \to \sim p$, the statement is proved. (With the wrong negation the contrapositive would claim $m + n$ is even whenever one of them is odd, and $1 + 2 = 3$ breaks it.)

b. Converse: "if m is even or n is even, then $m + n$ is odd". **False**: $m = 2$, $n = 4$ makes the hypothesis true and $m + n = 6$ even. (What is true: $m + n$ is odd $\leftrightarrow$ exactly one of m, n is even.)

MARKING SCHEME

✓ a. The contrapositive with the "or" negated to "and": m, n both odd $\to$ $m + n$ even.
✓ Its proof with $m = 2j + 1$, $n = 2k + 1$, and why it proves the original.
✓ b. The converse written correctly.
✓ False, with a counterexample such as $m = 2$, $n = 4$.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	0	12	13	**25**

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