Unit 1 · Logic and Set

ANSWER KEY · FOR TEACHERS

Full marks 41

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 4

1. statement
2. hypothesis
3. contrapositive
4. converse
5. tautology
6. contradiction
7. equivalent
8. De Morgan's

Each carries 0.5 mark.

2 True or false

U · 3

i. ✓ **True**

ii. ✗ **False**: It is an open sentence: true for $x = \pm 2$, false for every other x, so it has no truth value until x is given.

iii. ✗ **False**: The negation is "Sabina failed English or failed Nepali" (at least one): $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

iv. ✓ **True**

v. ✗ **False**: The contrapositive does; the converse need not ("if $x = 2$ then $x^2 = 4$" is true, its converse false for $x = -2$).

vi. ✓ **True**

Each carries 0.5 mark.

3 Multiple choice

i. **c.** both p and q are false — K · 1

An inclusive "or" needs only one true part, so it fails only when both parts are false.

ii. **b.** $q \to \sim p$ — U · 1

The converse swaps hypothesis and conclusion and changes nothing else: $q \to \sim p$. $\sim q \to p$ is the contrapositive.

iii. **c.** $p \wedge \sim q$ — U · 1

$p \to q \equiv \sim p \vee q$, and De Morgan turns its negation into $p \wedge \sim q$: the conditional fails only when p holds and q doesn't.

iv. **c.** 16 rows — U · 1

Four simple statements, p, q, r, s, so $2^4 = 16$ rows. ($\sim q$ is not a new statement: it is made from q.)

v. **b.** $\{6\}$ — A · 1

$A \cup B = \{1, 2, 3, 4, 5\}$, so its complement is $\{6\}$, equal to $\overline{A} \cap \overline{B} = \{4, 5, 6\} \cap \{1, 6\}$. $\{1, 4, 5, 6\}$ is $\overline{A \cap B}$.

vi. **c.** $A = B$ — HA · 1

Always $A \cap B \subseteq A \subseteq A \cup B$. If the ends are equal, A equals both, and so does B: $A = B$. (And if $A = B$, both sides are A.) With only $A \subseteq B$, the union is B and the intersection A, equal only when $A = B$.

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4 Short answers

a. MARKING SCHEME U · 2

✓ $\sim(p \to q) \equiv p \wedge \sim q$ used (not another "if … then").

✓ "A number is divisible by 6 and it is not even."

b. MARKING SCHEME U · 2

✓ $p \to q$ false (T $\to$ F).

✓ $\sim q \leftrightarrow p$ true (T $\leftrightarrow$ T).

c. MARKING SCHEME A · 3

✓ The columns $\sim p$, $\sim q$ right.

✓ The columns $p \to q$ and $\sim q \to \sim p$ right, and agreeing in every row.

✓ Proving the contrapositive proves the conditional.

d. MARKING SCHEME A · 2

✓ $n(A \cup B) = 63$.

✓ $n(B - A) = 18$.

e. MARKING SCHEME U · 3

✓ His working is right (both {1, 3, 4}).

✓ An example is a check, not a proof: it shows the law only for these sets.

✓ A proof for any element x, with De Morgan's law of logic.

5 Structured question

ANSWER A · 5

a.

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \to (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

True in some rows, false in others: **a contingency**.

b. It is true exactly when p and q have the same truth value (both true or both false): it is equivalent to $p \leftrightarrow q$.

MARKING SCHEME

✓ The columns $p \vee q$ and $p \wedge q$ right.

✓ The last column T, F, F, T.

✓ A contingency.

✓ True when p and q agree.

✓ Equivalent to $p \leftrightarrow q$.

6 Long question

ANSWER

a. $x \in A \cap (B \cup C) \Leftrightarrow x \in A$ and $x \in B \cup C$ (intersection)
$\Leftrightarrow x \in A$ and ($x \in B$ or $x \in C$) (union)
$\Leftrightarrow$ ($x \in A$ and $x \in B$) or ($x \in A$ and $x \in C$) (distributive law of logic)
$\Leftrightarrow x \in A \cap B$ or $x \in A \cap C \Leftrightarrow x \in (A \cap B) \cup (A \cap C)$ (intersection, union). The two sets have the same elements. ✓

b. $B \cup C = \{2, 4, 5, 6, 7, 8\}$, so $A \cap (B \cup C) = \{2, 4, 5\}$. $A \cap B = \{2, 4\}$, $A \cap C = \{4, 5\}$, and their union is {2, 4, 5}. Equal. ✓

c. $p \wedge (\sim p \vee q) \equiv (p \wedge \sim p) \vee (p \wedge q)$ (distributive) $\equiv c \vee (p \wedge q)$ (complement) $\equiv p \wedge q$ (identity). ✓

MARKING SCHEME

✓ a. The first step: $x \in A$ and $x \in B \cup C$, then "or" for the union.

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✓ The distributive law of logic used, with its name. **A · 8**
✓ Back to $x \in A \cap B$ or $x \in A \cap C$.
✓ So $x \in (A \cap B) \cup (A \cap C)$: both directions ($\Leftrightarrow$) shown.
✓ b. The left side $\{2, 4, 5\}$.
✓ The right side $\{2, 4, 5\}$, equal.
✓ c. The distributive law, giving $(p \wedge \sim p) \vee (p \wedge q)$.
✓ Complement then identity: $p \wedge q$.

7 Think

ANSWER **HA · 3**

Flipping one switch changes one of p, q and leaves the other. If they agreed, they now disagree, and if they disagreed, they now agree; so $p \leftrightarrow q$ always changes, and so does the light. With $p \wedge q$, when the top switch is down (q false) the light is off whatever the bottom switch does: flipping the bottom switch changes nothing, so it doesn't work.

MARKING SCHEME

✓ Flipping one switch changes exactly one of p, q.
✓ So agreement becomes disagreement or the other way round: $p \leftrightarrow q$, and the light, always change.
✓ $p \wedge q$ doesn't work: with q false, flipping p leaves the light off.

8 Activity: Columns that agree

(not marked)

WHAT STUDENTS SHOULD FIND

$\sim p \vee q$, $p \rightarrow q$ and $\sim(p \wedge \sim q)$ all read T, F, T, T: they are equivalent, three ways of saying the same thing. $q \rightarrow p$, the converse, reads T, T, F, T: it differs in rows 2 and 3, so it says something else.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	13	19	4	**41**

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