Unit 23 · Dynamics

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **Option**: Area 7 is Computational Methods **or** Mechanics, each school one; this unit and Unit 22 are Mechanics
- **CDC's grid (2078):** Mechanics (Units 22 and 23 together): 6 of 75 marks, one 1-mark choice and one 5-mark short question
- a short question asks when and where a particle is at rest, the distance it travels, or the time, height and speed of a body thrown, dropped, or sliding down a smooth plane
- **MCQ (1 mark)**
  - a from $v(x)$; a on a smooth plane; at rest for $x = t^2 - 4t + 3$; the signed area
  - back after $2u/g$; 10 m in 2 s $\rightarrow$ 40 m in 4 s; a retardation from 20 m/s in 40 m
  - distance for $v = 6 - 2t$; speed after 9.8 m at 30°; times down 30° and 60° planes; the last 44.1 m
- **Short (5 marks)**
  - $x = 2t^3 - 15t^2 + 36t + 5$: at rest, distance
  - thrown up from a building: greatest height, time, speed at the ground
  - a dropped ball meets one thrown up; up a smooth 30° plane and back
- **Long (school tests)**: chords from the top of a vertical circle all take $\sqrt{2d/g}$

Key terms

Particle: a body whose size we ignore.

Position x: the signed distance from a fixed point O, one way positive.

Displacement: change of position; **distance**: length of path covered, never negative.

Velocity $v = dx/dt$, its sign the direction; **speed** $|v|$.

Acceleration $a = dv/dt = d^2x/dt^2$; **retardation**: v and a of opposite signs (slowing).

Smooth plane: no friction; **normal reaction** R at right angles to it.

Formulas

$a = v\,dv/dx$	v known as a function of x (chain rule) **Use:** acceleration from position
$v = u + at,\ s = ut + \frac{1}{2}at^2$	a **constant** **Use:** uniform acceleration
$v^2 = u^2 + 2as,\ s = \frac{1}{2}(u + v)t$	the same **Use:** uniform acceleration
$H = u^2/(2g)$, back after $2u/g$	thrown up at u; $g = 9.8$ m/s² **Use:** gravity
$a = g\sin\theta,\ v = \sqrt{2gh}$	from rest, height h **Use:** smooth plane
$t = \sqrt{2l/(g\sin\theta)}$	length l, $h = l\sin\theta$ **Use:** time down

Motion in a straight line

- **at rest** where $v = 0$; **turns back** where v changes sign
- v, a same sign: speeding up; opposite: slowing

- **distance**: add the lengths between turning points; **displacement**: final − first position
  - $x = t^3 - 9t^2 + 24t$: $v = 3(t-2)(t-4)$, at rest at 2 s and 4 s
- **velocity–time graph**: slope = acceleration; signed area = **displacement**; all areas positive = **distance**
- from v back to x: integrate, a starting value fixes C

Uniform acceleration: derived

1. $dv/dt = a \to v = u + at$
2. $ds/dt = u + at \to s = ut + \frac{1}{2}at^2$
3. $v\,dv/ds = a \to \frac{1}{2}v^2 = as + C \to v^2 = u^2 + 2as$
4. $at = v - u$ in step 2 $\to s = \frac{1}{2}(u+v)t$

- retardation: a negative
- nth second: $u + (a/2)(2n-1)$
- two unknowns: two facts, two equations together
- the graph a straight line: distance = area of triangles and trapeziums

Under gravity

- $g = 9.8$ m/s^2, down, air ignored; every body the same
- **up positive, and keep it**: $a = -g$ going up, at the top, coming down; below the start $s < 0$

Thrown up at u

1. top: $v = 0$ after u/g
2. greatest height $u^2/(2g)$
3. back at the start after $2u/g$, twice the time up
4. speed then u, downward

- let go from a moving body (a rising balloon): starts with that body's velocity
- two bodies meet: same position at the same t

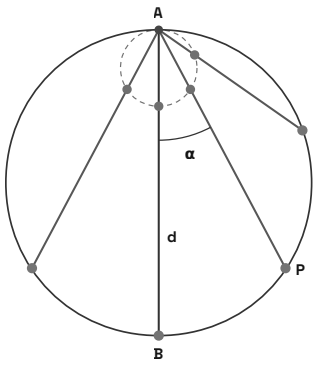
The well: splash heard after 2.0 s

- all falling: $h = 4.9 \times 2.0^2 = 19.6$ m
- with sound at 340 m/s: $\sqrt{2h/9.8} + h/340 = 2.0$; put $y = \sqrt{h}$ → a quadratic → **h ≈ 18.5 m**
- fall 1.95 s, sound 0.05 s; ignoring sound about 6% too deep

Smooth inclined plane

1. weight resolved: $mg\sin\theta$ down the slope, $mg\cos\theta$ into it
2. across: $R = mg\cos\theta$, they cancel
3. along: $ma = mg\sin\theta \to a = g\sin\theta$, the same for every mass

- $\theta = 90°$: free fall; 0°: nothing
- speed at the bottom $\sqrt{2gh}$: as a free fall through the same height, whatever the slope
- **up** the plane at u: retarded by $g\sin\theta$, goes $u^2/(2g\sin\theta)$, slides back
- only **smooth**: on a real slope friction takes some away



- wires from the top A of a vertical circle
- wire at α to AB: length $d\cos\alpha$ (angle in a semicircle), $a = g\cos\alpha$
- time $\sqrt{2d/g}$ on every wire

COMMON MISTAKES

✗ $v = u + at$ used with a changing a
✓ only for constant a; else calculus.

✗ distance = displacement
✓ add the lengths between turning points.

✗ up positive, but $a = +g$ on the way up
✓ $a = -g$ throughout.

✗ at the top $a = 0$
✓ $v = 0$; a is still $-g$.

✗ a stone let go from a rising balloon starts from rest
✓ it starts with the balloon's velocity.

✗ $a = g\cos\theta$ down the plane
✓ $g\sin\theta$, θ to the horizontal.

✗ heavier bodies slide faster
✓ $a = g\sin\theta$ for every mass.