Unit 22 · Statics

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **Option**: Area 7 is Computational Methods **or** Mechanics, each school one; this unit and Unit 23 are Mechanics
- **CDC's grid (2078):** Mechanics (Units 22 and 23 together): 6 of 75 marks, one 1-mark choice and one 5-mark short question
- a short question asks for the resultant of two forces (or the forces from their resultant), a resolution into two given directions, or the resultant of several forces at a point
- **MCQ (1 mark)**
  - the formula for R; the resolved parts
  - a resultant that can't be (7 N and 4 N); equal forces at right angles; the bisector
  - greatest at 0°; equal to either force at 120°; 3 N and 5 N at 60°
  - forces east, north and west; R at right angles to P; 3, 4, 5 N in equilibrium
- **Short (5 marks)**
  - $R = \sqrt{3}\,Q$ at 30° to $P \rightarrow P = Q$ or $2Q$
  - resolve 100 N at 60° and 45° either side
  - 10, 20, 30 N at 0°, 120°, 240°
- **Long (school tests)**: Q doubled doubles R; Q reversed doubles it again

Key terms

Force: a push or a pull; a vector with a **point of application**, its **line of action** through it.

Units: newton (N); **kilogram weight** (kg wt) ≈ 9.8 N; use the question's unit.

Coplanar: lines of action in one plane. **Concurrent**: through one point (forces at a point).

Resultant: the single force with the same effect as all of them; they are its **components**.

Composition: finding the resultant; **resolution**: splitting one force into components.

Equilibrium: resultant zero.

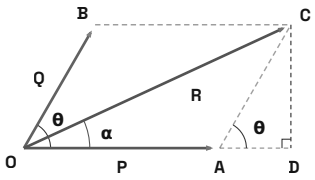
Like forces: the same way, add; **unlike**: opposite ways, subtract.

Formulas

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$	θ between P and Q **Use:** parallelogram law
$\tan\alpha = Q\sin\theta/(P + Q\cos\theta)$	α from P **Use:** direction
$R = 2P\cos(\theta/2)$	$Q = P$; bisects the angle **Use:** equal forces
$F_1 = F\sin\beta/\sin(\alpha + \beta)$	F_1 at α to F, F_2 at β **Use:** resolution
$F_2 = F\sin\alpha/\sin(\alpha + \beta)$	the other part **Use:** resolution
$X = \Sigma F\cos\theta$, $Y = \Sigma F\sin\theta$, $R = \sqrt{X^2 + Y^2}$	$\tan\alpha = Y/X$ **Use:** many forces

Parallelogram law

Statement: two forces at a point represented in magnitude and direction by two adjacent sides of a parallelogram drawn from that point have their resultant represented in magnitude and direction by the diagonal through that point.



- found by experiment
- $AC = Q$, $\angle CAD = \theta$
- $AD = Q\cos\theta$, $CD = Q\sin\theta$
- $OC^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$
- θ obtuse: AD negative, formulas still hold; $P + Q\cos\theta = 0 \to \alpha = 90°$
- **range**: $|P - Q| \le R \le P + Q$; greatest at $\theta = 0°$, least at 180°
- **equal forces**: $R^2 = 4P^2\cos^2(\theta/2)$; $\tan\alpha = \tan(\theta/2)$
  - $R = P$ when $\theta = 120°$; at 90°, $R = P\sqrt{2}$
- **right angles**: $R = \sqrt{P^2 + Q^2}$, $\tan\alpha = Q/P$
- forces from their resultants: two equations in P, Q

Resolution

- one force → two components with the same effect, along directions that matter (a slope, a road)
- **any two directions**: triangle of F_1, F_2, F; angles α, β, $180° - (\alpha + \beta)$; sine rule
  - each component carries the sine of the **other** angle: the closer direction takes more
- **at right angles**: $F\cos\alpha$ and $F\sin\alpha$, the **resolved parts**
- at θ to OX: $F\cos\theta$ along OX, $F\sin\theta$ along OY
- weight W on a slope θ: $W\sin\theta$ down it, $W\cos\theta$ into it

Many forces at a point

1. resolve each along OX and OY
2. add with signs: X, Y
3. $R = \sqrt{X^2 + Y^2}$, $\tan\alpha = Y/X$

- **quadrant** from the signs of (X, Y):
  - $X < 0$, $Y > 0$: $180° - \tan^{-1}|Y/X|$
  - $X < 0$, $Y < 0$: $180°$ + it; $X > 0$, $Y < 0$: $360°$ − it
  - $X = 0$: 90° or 270°
  - $X = -4$, $Y = 3$: 143.1°, not −36.9°
- **equilibrium** ⇔ $X = 0$ and $Y = 0$; head to tail the forces close a polygon
- three forces in equilibrium (triangle of forces, Lami): Grade 12

The chariot: two ropes, 4000 N each, θ apart: $R = 8000\cos(\theta/2)$

θ	0°	40°	60°	90°	120°
R (N)	8000	7518	6928	5657	4000

- a third wasted at $\cos(\theta/2) = ⅔$: $\theta \approx$ **96.4**°
- under a tenth lost: within about 52°
- the sideways parts only cancel each other

COMMON MISTAKES

✗ 3 N and 4 N always make 7 N
✓ anything from 1 N to 7 N.

✗ $R^2 = P^2 + Q^2 - 2PQ\cos\theta$
✓ plus: θ is the angle between the forces.

✗ α measured from Q
✓ the formula gives α from P.

✗ resolving with the sine of its own angle
✓ each takes the sine of the other angle.

✗ $\tan\alpha = Y/X$, first answer taken
✓ the quadrant of (X, Y).

✗ equilibrium when $X = 0$ only
✓ both $X = 0$ and $Y = 0$.