# Unit 16 · Probability

STUDY NOTES

### WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2078):** Statistics and Probability (Units 15 and 16 together): 6 of 75 marks, one 1-mark choice and one 5-mark short question
- **CDC's model paper:** a higher-ability choice, a student's chance when three students solve a problem with probability ¾; in a short question shared with skewness, the probability that a leap year has 53 Sundays (2 marks)
- **MCQ (1 mark)**
  - what can't be a probability; an empirical probability
  - at least one head; red or an ace; from odds
  - exhaustive but not exclusive; $P(A) = 2P(B) = 3P(C)$
  - two seeds, at least one sprouts; 53 Sundays; three solvers
- **Short (5 marks, in parts)**
  - 53 Mondays or Tuesdays; a multiple of 3 or of 5
  - independent $A$, $B$: $P(A \cap B)$, $P(A \cup B)$, neither, exactly one
  - $A$, $B$ independent $\Rightarrow$ $A$, $\overline{B}$ independent
- **Long (school tests)**: two dice; show two events independent; $P(C \cup D)$ and whether independent

## Key terms

**Random experiment**: outcomes we can list but not predict.

**Sample space** $S$: the set of all outcomes; an **event** is a subset; its outcomes are the **favorable cases**.

**Equally likely**: none has a better chance than another (a fair die; not a seed sprouting).

**Mutually exclusive**: can't happen together, $A \cap B = \emptyset$.

**Exhaustive**: together cover every outcome, $A \cup B \cup \ldots = S$.

**Independent**: one happening doesn't change the other's chance; else **dependent**.

**Odds in favor** $m : (n - m)$; **odds against** $(n - m) : m$.

## Formulas

| | |
|---|---|
| $P(E) = m/n = n(E)/n(S)$ | exhaustive, exclusive, equally likely cases **Use:** classical |
| $P(E) = \lim_{N \to \infty} r/N$ | $r$ times in $N$ trials **Use:** empirical |
| $P(\overline{E}) = 1 - P(E)$ | the $n - m$ cases against **Use:** complement |
| $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ | exclusive: $P(A) + P(B)$ **Use:** addition law |
| $P(A \cap B) = P(A)P(B)$ | the rule **and** the test **Use:** independent events |
| $1 - P(\overline{A_1})P(\overline{A_2}) \cdots$ | independent events **Use:** at least one |

## Two definitions

### Classical

- $0 \le P(E) \le 1$; $P(S) = 1$ certain; $P(\emptyset) = 0$ impossible
- two dice: 36 equally likely throws

### Empirical

- no equally likely cases (a seed, a cable, a baby) → repeat the trial
- relative frequency $r/N$ settles as $N$ grows; a large $N$ estimates
- coin: 260 heads in 500 → about 0.52
- where both can be used (a die), they agree

## The two laws

### Addition

- the outcomes in both are counted twice in $P(A) + P(B)$
- mutually exclusive in pairs: add straight
- red or an ace: $26/52 + 4/52 - 2/52 = 28/52$

### Multiplication (independent)

- independent ⇔ $P(A \cap B) = P(A)P(B)$; dependent when it fails
- many events: the law for every group of them
- $A$, $B$ independent → so are $A$ and $\overline{B}$, $\overline{A}$ and $\overline{B}$
  - $P(A \cap \overline{B}) = P(A) - P(A \cap B) = P(A)(1 - P(B))$
- exactly one: $P(A)P(\overline{B}) + P(\overline{A})P(B)$

### Independent is not mutually exclusive

- test with the multiplication law, never by whether they overlap
- "sum 10+" and "sum 7": no common throw → exclusive, and dependent

**Leap year**: 366 days = 52 weeks + 2 days; the 2 days: 7 equally likely pairs

- 53 Sundays: Sat–Sun or Sun–Mon → 2/7
- 53 Mondays or Tuesdays: Sun–Mon, Mon–Tue, Tue–Wed → 3/7

**Three solvers** (½, ⅓, ¼ alone): solved = $1 - ½ \cdot ⅔ \cdot ¾ = ¾$; exactly one: 11/24 (multiply within a case, add exclusive cases)

**The maize field**: each seed 0.85, independent (our assumption); 3,900 spots a ropani

| Seeds a spot | P(spot empty) | Empty spots |
|---|---|---|
| 1 | 0.15 | 585 |
| 2 | 0.0225 | about 88 |
| 3 | 0.003375 | about 13 |

- two seeds: few enough to fill at thinning; a third saves about 75 spots for 3,900 seeds
- one spot shares soil, water, pests → not quite independent: expect more empty spots

## For the engineering entrance (IOE)

- **conditional**: $P(B \mid A) = P(A \cap B)/P(A)$; $P(A \cap B) = P(A)P(B \mid A)$ for any events
- independent ⇔ $P(B \mid A) = P(B)$
- two aces without replacement: $4/52 \cdot 3/51 = 1/221$
- **total probability**: $P(A) = \Sigma\, P(E_i)P(A \mid E_i)$, $E_i$ exclusive and exhaustive
- **Bayes**: $P(E_i \mid A) = P(E_i)P(A \mid E_i) \,/\, \Sigma\, P(E_j)P(A \mid E_j)$
  - seed 60% $F_1$ (90%), 40% $F_2$ (80%); a failure: $P(F_2 \mid \text{fail}) = 0.08/0.14 = 4/7$
- **binomial**: $P(X = r) = {}^nC_r\, p^r\, q^{n-r}$, $q = 1 - p$, $r = 0, 1, \ldots, n$
- mean $np$, variance $npq$ (so variance < mean)
- two seeds: none 0.0225, one 0.255, both 0.7225

**COMMON MISTAKES**

✗ $P(A \cup B) = P(A) + P(B)$ always
✓ subtract $P(A \cap B)$ unless exclusive.

✗ mutually exclusive, so independent
✓ exclusive events (both possible) are dependent.

✗ $P(A \cap B) = P(A)P(B)$ for any events
✓ only independent ones.

✗ at least one by adding the chances
✓ 1 − the product of the failures.

✗ odds against 5 : 3 → $P = 5/8$
✓ 3/8.

✗ a probability above 1 or negative
✓ always in [0, 1].

✗ leap year: 1 extra day
✓ 2 extra days.