Unit 11 · Straight Line

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2078):** Analytic geometry (Units 11–13 together): 9 of 75 marks; with trigonometry and vectors it shares three 1-mark choices, two 5-mark short questions and one 8-mark long question
- **CDC's model paper:** a choice, the bisectors of the angles between $x = 2$ and $y = 2$; in a long question, the bisector of the angle containing the origin (3 marks)
- **MCQ (1 mark)**
  - the perpendicular's formula and a value; distance between parallel lines
  - the two bisectors (perpendicular to each other); points equally far from two parallel lines
  - k from a distance; a square's area from two sides; the foot of a perpendicular
- **Short (5 marks, in parts)**
  - a perpendicular's length; distance between parallel lines (make a, b match)
  - lines parallel to a given one at a given distance from a point
  - both bisectors; which is acute, which contains the origin
  - product of two perpendiculars is b^2
- **Long (8 marks)**
  - a triangle: perpendicular from the origin, a bisector, the incenter and its distance from the sides

Formulas

$d = \dfrac{\lvert ax_1 + by_1 + c\rvert}{\sqrt{a^2 + b^2}}$	(x_1, y_1) the point; line $ax + by + c = 0$ **Use:** point to line
$d = \dfrac{\lvert c\rvert}{\sqrt{a^2 + b^2}}$	the p of the normal form **Use:** origin to line
$d = \dfrac{\lvert c_1 - c_2\rvert}{\sqrt{a^2 + b^2}}$	same a, b in both **Use:** between parallel lines

Length of the perpendicular

- **distance** from a point to a line = the perpendicular PN (foot N): any other PQ is a hypotenuse, longer
- in words: put the point in the left side, take its size, divide by $\sqrt{a^2 + b^2}$

Derivation

1. PN has slope $b/a \to N = (x_1 + ta, y_1 + tb)$
2. N on the line $\to t = -(ax_1 + by_1 + c)/(a^2 + b^2)$
3. $PN = |t|\sqrt{a^2 + b^2}$

Which side?

- $ax + by + c$ is 0 on the line, positive on one side, negative on the other
- same sign at two points: same side; different signs: opposite sides
- origin gives c: on the origin's side when the value has the sign of c

Parallel lines

- write both with the same a, b first
- why: a point of the first has $ax + by = -c_1$; its distance from the second

Bisectors of the angles

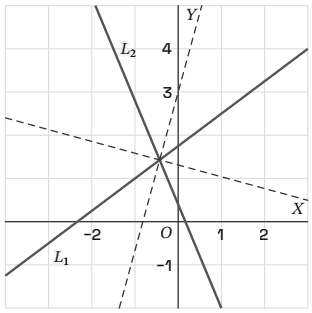
- two crossing lines: **two bisectors**, one per pair of opposite angles
- a point on a bisector is **equally far** from both lines (congruent right triangles); and the other way round

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

1. equal distances (sizes of the two left sides over their roots)
2. equal sizes: equal, or one minus the other
3. each sign a line: the two bisectors

- the bisectors are **perpendicular**: half of 180°
- **containing the origin**: make $c_1 > 0$, $c_2 > 0$; then the **+** sign

- **acute**: angle θ between a bisector and one line; $\tan\theta < 1$ (under 45°) → it bisects the acute angle
- by a sign (IOE): with $c_1, c_2 > 0$, $a_1a_2 + b_1b_2 < 0$ → + is acute; > 0 → + is obtuse



two lines and their two bisectors, at right angles

The highway: center line $3x - 4y = 0$ (through (0, 0) and (80, 60)), $\sqrt{9 + 16} = 5$

- $F(20, 40)$: $|60 - 160|/5 = 20$ m → 5 m inside the 25 m boundary
- $G(10, 45)$: $|30 - 180|/5 = 30$ m → 5 m outside
- both values negative: same side
- boundary on that side: $3x - 4y + 125 = 0$

For the engineering entrance (IOE)

- **foot** $N(h, k)$: $(h - x_1)/a = (k - y_1)/b = -(ax_1 + by_1 + c)/(a^2 + b^2)$
- **image**: the same with $-2(ax_1 + by_1 + c)/(a^2 + b^2)$; the foot is the midpoint of P and its image
- (1, 2) in $x - y + 3 = 0$: foot (0, 3), image (−1, 4)

Centers of a triangle (sides a, b, c opposite A, B, C)

- **centroid** (medians): $\left((x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3\right)$
- **incenter** (angle bisectors; equally far from the sides): $x = (ax_1 + bx_2 + cx_3)/(a + b + c)$, y the same with y_1, y_2, y_3
- **circumcenter** (perpendicular bisectors; equally far from the vertices): solve two together
- **orthocenter** (altitudes): solve two altitudes; a right triangle: the right-angled vertex
- $O(0, 0)$, $A(12, 0)$, $B(0, 5)$: incenter (2, 2); circumcenter (6, 5/2), the hypotenuse's midpoint

COMMON MISTAKES

✗ distance without the size bars
✓ always $|\ldots|$: a distance is never negative.

✗ parallel lines with different a, b
✓ make them match before $|c_1 - c_2|$.

✗ one bisector only
✓ two, with ±.

✗ + sign always contains the origin
✓ only after making $c_1, c_2 > 0$.

✗ the + bisector is always acute
✓ test with $\tan\theta < 1$ or the sign of $a_1a_2 + b_1b_2$.

✗ dividing by $a^2 + b^2$
✓ by its square root.