Unit 2 · Real Numbers

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2078):** Algebra (Units 1–8 together): 20 of 75 marks
  - two 1-mark choices
  - two 5-mark short questions
  - one 8-mark long question
- **MCQ (1 mark)**
  - what an interval contains; the inequality for an interval
  - $\sqrt{x^2}$; a distance on the line
  - which property holds for all x, y
  - integers in $|x - c| < r$; $|x + 3| = |x - 5|$
- **Short (5 marks, in parts)**
  - intervals on a number line: union, intersection, difference, complement
  - solve an absolute-value inequality; answer as an interval, on a line
  - prove an inequality ($|x - y| \geq |x| - |y|$)
  - an equation such as $|x - 2| = 3x + 6$: check each root
- **Long (8 marks)**
  - two absolute values: split the line; explain with distances

Key terms

Real numbers $\mathbb{R}$: rational and irrational together; $\mathbb{N} \subset W \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$.

Rational: p/q, $q \neq 0$; decimal **ends** ($3/8 = 0.375$) or **repeats** ($2/11 = 0.1818\ldots$).

Irrational: decimal neither ends nor repeats ($\sqrt{2}$, π).

Real line: each real number one point, each point one real number.

Dense: between any two rationals an irrational, and the other way round.

Completeness: rationals and irrationals together fill the line with no gaps.

Ordered: exactly one of $a < b$, $a = b$, $a > b$; $a < b$: a left of b.

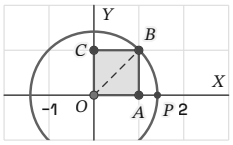
Interval: all real numbers between two ends, no gap.

Bounded / unbounded: both ends finite / an end ∞ or $-\infty$.

Absolute value (modulus) $|x|$: distance of x from 0.

Neighborhood of c: $(c - r, c + r)$, the numbers within r of c.

Tolerance: the radius in "size ± tolerance".

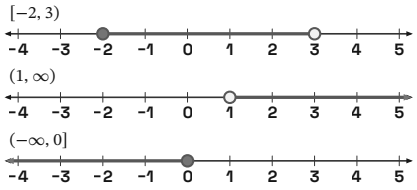


- square of side 1: diagonal $OB = \sqrt{2}$
- compass about O through $B \rightarrow$ point $P = \sqrt{2}$ on the line
- a real point no fraction names

Intervals

Interval	Set	Name
$[a, b]$	$a \leq x \leq b$	closed
(a, b)	$a < x < b$	open
$[a, b)$	$a \leq x < b$	half-open
$(a, b]$	$a < x \leq b$	half-open
(a, ∞)	$x > a$	unbounded
$[a, \infty)$	$x \geq a$	unbounded
$(-\infty, b)$	$x < b$	unbounded
$(-\infty, b]$	$x \leq b$	unbounded
$(-\infty, \infty)$	$\mathbb{R}$	the whole line

- square bracket: end **in**, filled dot; round: end **out**, open dot
- ∞ is **not a number**: always a round bracket, no dot, an arrow



$[-2, 3)$, $(1, \infty)$, $(-\infty, 0]$

- intervals are sets: union, intersection on one number line
- $A = [-2, 3)$, $B = (1, \infty)$
  - $A \cup B = [-2, \infty)$
  - $A - B = [-2, 1]$
  - $A \cap B = (1, 3)$
  - $\overline{A} = (-\infty, -2) \cup [3, \infty)$
- complement turns every end's bracket round

Absolute value

$$|x| = x \text{ if } x \geq 0; \qquad |x| = -x \text{ if } x < 0$$

- $-x$ is positive when x is negative: $-(-5) = 5$
- $|a - b| = |b - a|$ = **distance between** a and b: -2 to 5 is 7
- $\sqrt{x^2} = |x|$, not x

Properties (all real x, y)

- $|x| \geq 0$; $|x| = 0$ only for $x = 0$
- $|-x| = |x|$
- $-|x| \leq x \leq |x|$
- $|xy| = |x||y|$; $|x/y| = |x|/|y|$, $y \neq 0$
- **triangle inequality** $|x + y| \leq |x| + |y|$; equal when same sign (or one is 0)

Triangle inequality, proof

1. $-|x| \leq x \leq |x|$, $-|y| \leq y \leq |y|$
2. add: $-(|x| + |y|) \leq x + y \leq |x| + |y|$
3. between $-k$ and $k \to |x + y| \leq k = |x| + |y|$

- $|x - y| \geq |x| - |y|$: same idea, from $|x| = |(x - y) + y| \leq |x - y| + |y|$

Equations and inequalities

THE KEY THEOREM, FOR A > 0

- $|x| < a \Leftrightarrow -a < x < a$: the interval $(-a, a)$
- $|x| > a \Leftrightarrow x < -a$ or $x > a$: the two rays outside
- with $\leq, \geq$ the ends come in
- $|x - c| < r \Leftrightarrow c - r < x < c + r$
  - $x \in (c - r, c + r)$, within r of c
- $|x - c| > r \Leftrightarrow x \in (-\infty, c - r) \cup (c + r, \infty)$

- **equation** $|2x - 3| = 5$: $2x - 3 = 5$ or $-5 \to x = 4$ or -1; as distance: x is $5/2$ from $3/2$
- **inequality** $|3x - 2| \leq 4$: $-4 \leq 3x - 2 \leq 4 \to [-2/3, 2]$
- $|x + 1| > 2$: $x < -3$ or $x > 1 \to (-\infty, -3) \cup (1, \infty)$
- $|x - 2| = 3x + 6$ (a variable side): solve both cases, then **check** each in the original

Several absolute values

1. split the line where each inside is 0
2. on each piece, drop the bars with the right sign
3. solve; keep only answers inside their piece
4. as distances: $|x - 1| + |x + 2|$ = distance to 1 + distance to -2; between them always 3; $= 5$ at -3 and 2

- $|x| < -2$: no solution; $|x| > -2$: every x
- football: $|c - 69| \leq 1 \Leftrightarrow 68 \leq c \leq 70$: one interval by its **ends** or by its **center and radius**; weight 410–450 g is $|w - 430| \leq 20$

COMMON MISTAKES

✗ ∞ in a square bracket
✓ always round: ∞ is not a number.

✗ $\sqrt{x^2} = x$
✓ $\sqrt{x^2} = |x|$.

✗ $|a + b| = |a| + |b|$
✓ only for the same sign: $|-3 + 5| = 2$, not 8.

✗ $|x| > a$ gives $-a < x < a$
✓ that is $|x| < a$; $|x| > a$ is two rays.

✗ $|x| < -2$ solved as usual
✓ no solution: a distance is never negative.

✗ an answer outside its piece kept
✓ check each answer lies in its piece.

✗ complement keeps the brackets
✓ turn every end round.