Unit 1 · Logic and Set

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2078):** Algebra (Units 1–8 together): 20 of 75 marks
  - two 1-mark choices
  - two 5-mark short questions
  - one 8-mark long question
- **MCQ (1 mark)**
  - a connective's truth value (when is $p \vee q$ false?)
  - converse, inverse, contrapositive of a conditional
  - $\sim(p \to q)$, a tautology or a contradiction
  - a law of sets: $A \cap \overline{A}$, $A - B$, De Morgan with listed sets
- **Short (5 marks, in parts)**
  - write compound statements in words; their truth values
  - a truth table; tautology or not
  - prove an equivalence by truth table
  - prove a set identity
- **Long (8 marks)**
  - simplify with the laws of logic, naming each law
  - prove a set law and show it with Venn diagrams

Key terms

Statement (proposition): a sentence either true or false, not both; its **truth value** T or F.

Not statements: commands, questions, exclamations, **open sentences** ($x + 3 = 7$: true only for some x).

Simple / compound: no smaller statement inside / simple statements joined by connectives.

Logical connectives: not, and, or, if … then, if and only if.

Hypothesis (antecedent) and **conclusion** (consequent): p and q in $p \to q$.

Tautology: true in every row; **contradiction**: false in every row; **contingency**: sometimes true, sometimes false.

Logically equivalent $X \equiv Y$: same truth value in every row; then $X \leftrightarrow Y$ is a tautology.

Inclusive or: "or" allows both; **exclusive or** $\sim(p \leftrightarrow q)$: exactly one.

Connectives

Name	Symbol	Read	True when
negation	$\sim p$	not p	p false
conjunction	$p \wedge q$	p and q	both true
disjunction	$p \vee q$	p or q	at least one true
conditional	$p \to q$	if p then q	all but T → F
biconditional	$p \leftrightarrow q$	p iff q	same truth value

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

- other notations: $\neg p$, p'; $p \Rightarrow q$, $p \Leftrightarrow q$
- negation flips the value: "Nepal has 75 districts" is **not** the negation of "Nepal has 77 districts"
- $p \to q$ as a promise: broken only when p true, q false; **true whenever p is false**
- $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

From $p \to q$

Name	Form	Same value as $p \to q$?
converse	$q \to p$	not always
inverse	$\sim p \to \sim q$	not always
contrapositive	$\sim q \to \sim p$	always

- "divisible by 4 → even": converse and inverse false (6), contrapositive true
- contrapositive: negate both and swap; converse: swap only

Truth tables

1. n simple statements → 2^n rows (2 → 4, 3 → 8, 4 → 16)
2. one column per part, from the inside out
3. last column: the whole statement
4. all T: tautology; all F: contradiction; two columns alike: equivalent

- $(p \vee q) \wedge \sim p$: T only when p F, q T
- $p \to q \equiv \sim p \vee q \equiv \sim q \to \sim p$: a proof by contrapositive is a full proof

- $\sim(p \to q) \equiv p \wedge \sim q$: hypothesis holds, conclusion fails (not another "if … then")
- tautologies: $p \vee \sim p$; $[p \wedge (p \to q)] \to q$ (**modus ponens**); $[(p \to q) \wedge (q \to r)] \to (p \to r)$ (**chain rule**, hypothetical syllogism); $(p \wedge q) \to p$
- contradiction: $p \wedge \sim p$; $(p \to q) \wedge (p \wedge \sim q)$
- not equivalent: one row where the columns differ is enough
- $(p \wedge q) \to r \equiv p \to (q \to r)$

Laws of logic and of sets

t always true, c always false; for sets read $\wedge$ as $\cap$, $\vee$ as $\cup$, $\sim$ as the complement, t as U, c as $\emptyset$.

Law	Logic	Sets
idempotent	$p \wedge p \equiv p, p \vee p \equiv p$	$A \cup A = A, A \cap A = A$
commutative	$p \wedge q \equiv q \wedge p$	$A \cup B = B \cup A$
associative	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(A \cup B) \cup C = A \cup (B \cup C)$
distributive	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
De Morgan's	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
	$\sim(p \vee q) \equiv \sim p \wedge \sim q$	$\overline{A \cup B} = \overline{A} \cap \overline{B}$
identity	$p \wedge t \equiv p, p \vee c \equiv p$	$A \cap U = A, A \cup \emptyset = A$
complement	$p \wedge \sim p \equiv c, p \vee \sim p \equiv t$	$A \cap \overline{A} = \emptyset, A \cup \overline{A} = U$
double negation	$\sim(\sim p) \equiv p$	difference: $A - B = A \cap \overline{B}$

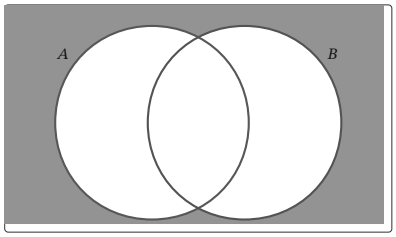
- De Morgan: negating swaps $\wedge$ and $\vee$ and negates each part ("not both" = "at least one fails")
- simplifying: $p \vee (\sim p \wedge q) \equiv (p \vee \sim p) \wedge (p \vee q)$ (distributive) $\equiv t \wedge (p \vee q)$ (complement) $\equiv p \vee q$ (identity)

Sets as logic

Sets	Means	Logic
$x \in A \cup B$	$x \in A$ or $x \in B$	$\vee$
$x \in A \cap B$	$x \in A$ and $x \in B$	$\wedge$
$x \in \overline{A}$	not $x \in A$	$\sim$
$x \in A - B$	$x \in A$ and $x \notin B$	$p \wedge \sim q$
$A \subseteq B$	if $x \in A$ then $x \in B$	$\to$
$A = B$	$x \in A$ iff $x \in B$	$\leftrightarrow$

Proving $X = Y$

1. any $x \in X \to$ show $x \in Y$ ($X \subseteq Y$)
2. any $x \in Y \to$ show $x \in X$ ($Y \subseteq X$)
3. steps that read both ways: one chain of $\Leftrightarrow$, each step with its reason (a definition, or the matching law of logic)



$\overline{A \cup B} = \overline{A} \cap \overline{B}$: outside both circles

- **De Morgan**: $x \in \overline{A \cup B} \Leftrightarrow x \notin A \cup B \Leftrightarrow$ not ($x \in A$ or $x \in B$) $\Leftrightarrow x \notin A$ and $x \notin B \Leftrightarrow x \in \overline{A} \cap \overline{B}$
- **distributive**: $x \in A \cap (B \cup C) \Leftrightarrow x \in A$ and ($x \in B$ or $x \in C$) $\Leftrightarrow$ ($x \in A \cap B$) or ($x \in A \cap C$)
- **counting**: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$; $A - B, A \cap B, B - A$ share nothing; the overlap is counted twice
- $A - (B \cup C) = (A - B) \cap (A - C)$: repeat "$x \in A$" ($p \wedge p \equiv p$)
- $A - (A - B) = A \cap B$; $A \cup (\overline{A} \cap B) = A \cup B$
- $A \cup B = A \cap B$ exactly when $A = B$
- listed sets agreeing: a **check**, not a proof; one example disagreeing **disproves** an identity

Staircase light (two switches): on when both up or both down: $p \leftrightarrow q$; wired the other way, $\sim(p \leftrightarrow q)$ (exclusive or). As sets: $(P \cap Q) \cup (\overline{P} \cap \overline{Q})$.

- AND gate = switches in series; OR gate = in parallel; 1 for T, 0 for F
- $p \to q$ from gates: p through NOT into OR with q

COMMON MISTAKES

✗ $p \to q$ false when p is false
✓ true whenever p is false; false only for T $\to$ F.

✗ "or" means one but not both
✓ inclusive: both allowed.

✗ $\sim(p \wedge q) \equiv \sim p \wedge \sim q$
✓ $\sim p \vee \sim q$: "and" becomes "or".

✗ converse = contrapositive
✓ converse $q \to p$; contrapositive $\sim q \to \sim p$.

✗ negation of $p \to q$ is $\sim p \to \sim q$
✓ $p \wedge \sim q$.

✗ 4 rows for 4 statements
✓ $2^4 = 16$ rows.

✗ $x^2 = 9$ is a statement
✓ open sentence until x is given.

✗ one example proves a set identity
✓ only an element-wise proof does.