Unit 5 · Modeling and Simulation

ANSWER KEY · FOR TEACHERS

Full marks 31

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. model
2. assumption
3. parameter
4. state
5. update rule
6. time step
7. stochastic
8. Monte Carlo

Each carries 0.5 mark. **K · 4**

2 Label the diagram

1. The real system
2. The model
3. The program
4. Run it

Each carries 0.5 mark. **U · 2**

3 Multiple choice

i. **b.** the traffic on a Tuesday morning is like the traffic on any other morning **U · 1**

An assumption is something the model takes to be true without checking it. The distances, the route and the stops are what the model keeps (its data). That every morning's traffic is like every other is assumed, and it could be wrong: festival days, strikes and rain all change it.

ii. **c.** about 0.34 m **A · 1**

For Euler's method the time-step error shrinks in proportion to the step: a step 10 times smaller gives an error about 10 times smaller. (The table above gave 0.22 m: "about" is the right word.) It never becomes exactly 0, however small the step.

iii. **c.** 75% **A · 1**

$1 - 1 \div R_0 = 1 - 1 \div 4 = 0.75$, so 75%. Then each infected person, who would infect 4 people if all could catch it, infects $4 \times 0.25 = 1$ at most, so the disease can't grow. Not everyone needs to be immune: the immune protect the rest.

iv. **c.** 100,000 **A · 1**

The error shrinks in proportion to $1 \div \sqrt{\text{trials}}$. To make it 10 times smaller, the square root of the trials must grow 10 times, so the trials must grow 100 times: from 1,000 to 100,000.

v. **c.** the hostel outbreak, where each day's meetings and recoveries are decided by random numbers **U · 1**

A stochastic model has chance in it, and gives a different result each run. The other three give the same numbers every time.

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4 An epidemic, one day

ANSWER A · 3

i. spread = 2 ÷ 5 = **0.4.** New cases = 0.4 × 990 × 10 ÷ 1,000 = 3.96. Recoveries = 10 ÷ 5 = 2. So S = 990 − 3.96 = **986.04**, I = 10 + 3.96 − 2 = **11.96**, R = 0 + 2 = **2.**

ii. $1 - 1 \div R_0 = 1 - 1 \div 2 =$ **50%.**

MARKING SCHEME

- ✓ spread = 0.4, new cases 3.96 and recoveries 2.
- ✓ S = 986.04, I = 11.96, R = 2.
- ✓ 50% ($1 - 1 \div R_0$).

5 Trace a simulation

ANSWER A · 4

t	y	vy	**Output**
0.0	0.0	15.0	0.0 0.0 15.0
0.5	7.5	10.0	0.5 7.5 10.0
1.0	12.5	5.0	1.0 12.5 5.0
1.5	15.0	0.0	1.5 15.0 0.0
2.0	15.0	-5.0	2.0 15.0 -5.0
2.5	12.5	-10.0	2.5 12.5 -10.0
3.0	7.5	-15.0	3.0 7.5 -15.0
3.5	0.0	-20.0	3.5 0.0 -20.0
			Below the ground at 4.0

ii. Each step moves the stone at the speed it had at the **start** of the step, as if it didn't change during the half second. Going up, the stone really slows during each step, so every step carries it too high: 15 m, not 11.25 m, and it lands late. This is the time-step error. A smaller step (0.01 s, say) brings the answer close to the formula's; the error shrinks about in proportion to the step.

MARKING SCHEME

- ✓ y 7.5, 12.5, 15.0, 15.0, 12.5, 7.5, 0.0 (each step adds `vy * dt` with the old vy).
- ✓ vy falling by 5 each step, from 15.0 to −20.0, and t by 0.5.
- ✓ The row for y = 0.0 printed (0 is not below the ground), then `Below the ground at 4.0`.
- ✓ The speed is taken as fixed during each step (time-step error); a smaller time step.

6 Short answer

MARKING SCHEME U · 2

- ✓ The model was fitted to those censuses, so matching them is no test; only data it wasn't fitted to can check it.
- ✓ Look for the assumption that stopped being true (a fixed growth rate, while births fell and many people went abroad), change the model, and check it again against new data.

7 What does it print?

MARKING SCHEME A · 2

- ✓ `1 18.5 True` and `2 20.0 True`: below 18 at the start, so the heater goes on, and the room warms by 1.5 a minute.
- ✓ `3 21.5 True`: 20.0 is not above 20, so the heater stays on; then `4 21.0 False`, off once the room is above 20.

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8 Find and fix the bug

MARKING SCHEME **A · 3**

- ✓ The seed inside the loop makes every trial the same throw.
- ✓ Line 6 removed, or the seed set once, before the loop.
- ✓ About 0.167 (1/6: 6 of the 36 throws add up to 7), not exactly the same each run.

9 Write a program

ANSWER **A · 4**

```
import random

trials = 100000
hits = 0
for i in range(trials):
    heads = 0
    for toss in range(5):
        if random.random() < 0.5:
            heads = heads + 1
    if heads >= 4:
        hits = hits + 1
print(hits / trials)
```

```
OUTPUT
0.18815
```

(A different number each run, close to the exact answer, 6 ÷ 32 = 0.1875: 5 ways to get 4 heads and 1 way to get 5, out of 32.) Also right: `random.randint(0, 1)` or `random.choice(["H", "T"])` for each toss.

MARKING SCHEME

- ✓ A loop of 100,000 trials.
- ✓ In each trial, 5 tosses, counting the heads.
- ✓ A trial with 4 or more heads counted as a hit.
- ✓ The share of hits printed (hits ÷ trials).

10 Think

ANSWER **HA · 2**

Any two, with their effect. The model assumes everyone meets everyone else equally; in real life people meet mostly their family and neighbors, which usually spreads the disease more slowly, so the real peak would be lower and later. It assumes people behave the same however bad it gets; in a real epidemic people stay home, wear masks and close schools, so the peak would be lower. It assumes nobody comes in from outside; new cases coming in would make it start sooner, and could make it higher. It assumes everyone is equally likely to catch it; a district with more old or weak people would need more beds than the model says.

MARKING SCHEME

- ✓ One assumption of the model named (everyone meets everyone equally; behavior doesn't change; nobody comes in; everyone equally likely to catch it or to need a bed), with its effect on the peak.
- ✓ A second assumption, with its effect.

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11 Activity: How big a step?

(not marked)

WHAT STUDENTS SHOULD FIND

Time step	Steps	Lands (Euler)	Off by
0.5 s	9	79.5 m	15.77 m
0.2 s	20	70.7 m	6.94 m
0.1 s	38	67.2 m	3.40 m
0.01 s	362	64.0 m	0.22 m

The formula gives 63.8 m. A step 10 times smaller gives an error about 10 times smaller, but takes about 10 times as many steps.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	4	6	19	2	**31**

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