Unit 11 · Force

ANSWER KEY · FOR TEACHERS

Full marks 49

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 4

1. periodic
2. oscillatory
3. effective length
4. centripetal
5. field intensity
6. potential
7. escape velocity
8. center of gravity

Each carries 0.5 mark.

2 Label the diagram

U · 2

1. mg cos θ
2. T
3. mg
4. mg sin θ back to the middle

Each carries 0.5 mark.

3 What pulls it to the center?

K · 3

1. a stone whirled on a string → **c.** the tension of the string
2. a bus going round a bend → **f.** friction between the tyres and the road
3. a satellite round the Earth → **g.** the Earth's gravitational pull
4. the electron round the nucleus (Bohr's model) → **e.** the electric attraction between them
5. clothes in a spinning washing machine drum → **d.** the push of the drum's wall
6. the Earth round the Sun → **b.** the Sun's gravitational pull

Each carries 0.5 mark.

4 Multiple choice

i. **c.** the mass of its bob — K · 1

$T = 2\pi\sqrt{(l/g)}$: it depends on the length and on g (which changes from place to place), but not on the bob's mass.

ii. **b.** twice as large — A · 1

$F = mv^2/r$: doubling v multiplies it by 4, doubling r divides it by 2, so overall it doubles.

iii. **c.** the acceleration due to gravity g — U · 1

$E = F/m = GM/R^2$, which is exactly g: 9.8 N/kg = 9.8 m/s^2.

iv. **b.** unstable equilibrium — U · 1

The slightest tilt lowers its center of gravity, and the line from G then misses the tip, so its weight turns it farther over: unstable.

v. **b.** 22.4 km/s — A · 1

$v_e = \sqrt{(2GM/R)} \propto \sqrt{M}$ at the same R. Four times the mass gives $\sqrt{4} = 2$ times the escape velocity: $2 \times 11.2 = 22.4$ km/s.

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5 Very short answers

a. MARKING SCHEME — K · 1

✓ 2 s (1 s each way) written.

b. MARKING SCHEME — K · 1

✓ Joule per kilogram (J/kg) written.

c. MARKING SCHEME — A · 1

✓ $T = 1/256\ \text{s} \approx 0.0039\ \text{s}$.

d. MARKING SCHEME — U · 1

✓ Neutral equilibrium written.

6 Short answers

a. MARKING SCHEME — U · 2

✓ The rod expands in summer, so the effective length l increases.

✓ $T = 2\pi\sqrt{(l/g)}$ increases, so each swing takes longer and the clock runs slow.

b. MARKING SCHEME — U · 2

✓ A vehicle on a bend needs a centripetal force; on a flat road only friction gives it, and on a wet road it may not be enough.

✓ On a banked road the road pushes on the tyres at a slant, and the inward part helps provide the centripetal force.

c. MARKING SCHEME — U · 2

✓ Potential is taken as zero at infinity.

✓ The Earth's attraction does the work as a mass comes in, so it has less energy than at infinity: $V = -GM/r$ is negative.

d. MARKING SCHEME — U · 2

✓ Centripetal force is a real force (tension, friction, gravity…) directed towards the center.

✓ Centrifugal force is a pseudo force, felt away from the center by someone going round with the body, of the same size mv^2/r.

7 Long answers

a. ANSWER — U · 4

Gravitational potential at a point is the work done in bringing a unit mass from infinity to that point; at the Earth's surface $V = -GM/R$.

Escape velocity is the smallest speed with which a body must be projected from the surface so that it escapes the Earth's gravitational field and never returns.

To escape, a body of mass m must climb from potential energy $mV = -GMm/R$ to zero at infinity, a gain of GMm/R, using its kinetic energy. Just escaping: $\frac{1}{2}mv_e^2 = GMm/R$, so $v_e = \sqrt{(2GM/R)}$. Since $V = -GM/R$, $\frac{1}{2}v_e^2 = -V$, or $v_e = \sqrt{(-2V)}$. The deeper (more negative) the potential, the greater the escape velocity.

MARKING SCHEME

✓ Gravitational potential: the work done in bringing a unit mass from infinity to a point ($V = -GM/R$ at the surface).

✓ Escape velocity: the smallest speed from the surface with which a body escapes the field and never returns.

✓ Just escaping, kinetic energy = the gain in potential energy: $\frac{1}{2}mv^2 = GMm/R$, so $v = \sqrt{(2GM/R)}$.

✓ With $V = -GM/R$: $\frac{1}{2}v^2 = -V$, or $v = \sqrt{(-2V)}$.

b. ANSWER

A body is stable while the vertical line through its center of gravity falls inside its base; once a tilt takes that line outside the base, it topples. So a body is more stable when its CG is low and its base is wide, because then it must be tilted farther before the line passes the edge ($\tan\theta = (b/2)/h$).

Racing cars are built low, which keeps their CG close to the road, and wide, which gives a wide base, so

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they can take sharp bends fast without overturning. Luggage piled on a bus's roof raises its CG, so a smaller tilt, on a bend or a sideways slope, is enough to overturn it. A · 4

MARKING SCHEME

- ✓ A body is stable while the vertical through its CG falls inside its base; outside it, the body topples.
- ✓ So a low CG and a wide base make it more stable, since it must be tilted farther (tan θ = (b/2)/h).
- ✓ Racing cars are low (low CG) and wide (wide base), so they don't overturn on sharp bends.
- ✓ Luggage on a bus's roof raises its CG, so a smaller tilt overturns it.

8 Numerical problems

a. **ANSWER** A · 2

T = 31.2 ÷ 20 = 1.56 s.

$$g = \frac{4\pi^2 l}{T^2} = \frac{4 \times 3.14^2 \times 0.60}{1.56^2} = \frac{23.66}{2.434} = 9.72 \text{ m/s}^2$$

g ≈ 9.7 m/s².

MARKING SCHEME

- ✓ T = 31.2 ÷ 20 = 1.56 s.
- ✓ g = $4\pi^2$l/T^2 = 9.7 m/s².

b. **ANSWER** A · 3

T = 1 ÷ 2 = 0.5 s; $v = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 1.0}{0.5} = 12.6$ m/s.

$$F = \frac{mv^2}{r} = \frac{0.5 \times 12.56^2}{1.0} = 78.9 \text{ N}$$

12.6 m/s; a tension of about 79 N.

MARKING SCHEME

- ✓ T = 0.5 s.
- ✓ v = 2πr/T = 12.6 m/s.
- ✓ F = mv^2/r ≈ 79 N.

c. **ANSWER** A · 4

GM = $6.67 \times 10^{-11} \times 6.4 \times 10^{23} = 4.27 \times 10^{13}$ N m²/kg; R = 3.4×10^6 m.

i. $E = \frac{GM}{R^2} = \frac{4.27 \times 10^{13}}{(3.4 \times 10^6)^2} = \frac{4.27 \times 10^{13}}{1.156 \times 10^{13}} =$ **3.7 N/kg**.

ii. $V = -\frac{GM}{R} = -\frac{4.27 \times 10^{13}}{3.4 \times 10^6} =$ **-1.26×10^7 J/kg**.

iii. $v_e = \sqrt{-2V} = \sqrt{2 \times 1.26 \times 10^7} = \sqrt{2.51 \times 10^7} =$ **5.0×10^3 m/s**, about 5.0 km/s.

MARKING SCHEME

- ✓ GM = 4.27×10^{13} N m²/kg, with R = 3.4×10^6 m.
- ✓ E = GM/R^2 = 3.7 N/kg.
- ✓ V = −GM/R = -1.26×10^7 J/kg.
- ✓ v = $\sqrt{(-2V)}$ = 5.0×10^3 m/s.

d. **ANSWER** A · 3

$$x = \frac{120 \times 50 + 30 \times 90}{120 + 30} = \frac{6000 + 2700}{150} = \frac{8700}{150} = 58 \text{ cm}$$

At the 58 cm mark, moved 8 cm from the middle towards the load.

MARKING SCHEME

- ✓ The stick's own weight acts at its 50 cm mark.
- ✓ x = (120 × 50 + 30 × 90) ÷ (120 + 30) written.
- ✓ x = 58 cm.

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9 Think

ANSWER HA · 3

$T = 2\pi\sqrt{(l/g)}$, so a smaller g gives a longer period, fewer swings in an hour, and a clock that runs slow. In Kathmandu, g is very slightly smaller, because it is farther from the Earth's center, so the clock runs slightly slow (by about 18 seconds a day: g falls by about 0.04 %, and T grows by half that, about 0.02 %; Kathmandu being a little farther north, and the rocks under the valley, change it a little). On the Moon, g is about a sixth of the Earth's, so the period is $\sqrt{6} \approx 2.4$ times as long and the clock runs very slow, about 2.4 times too slow.

MARKING SCHEME

- ✓ $T = 2\pi\sqrt{(l/g)}$: a smaller g gives a longer period, so the clock runs slow.
- ✓ In Kathmandu g is very slightly smaller (farther from the Earth's center), so it runs slightly slow.
- ✓ On the Moon g is about a sixth, so the period is about 2.4 times as long and it runs very slow.

10 Activity: Find g with a simple pendulum

(not marked)

WHAT STUDENTS SHOULD FIND

T^2 against l is a straight line through the origin, since $T^2 = (4\pi^2/g)\,l$. Its slope is about 4.0 s^2/m, which gives $g = 4\pi^2 \div 4.0 \approx 9.8$ m/s^2. Changing the bob, or the size of a small swing, doesn't move the points.

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **The Clay Owl** (exhibit): Count the owl's swings in 10 seconds, pushed gently and then harder: the time period depends on the string's length, not on the amplitude.
- **The Chime Instrument** (exhibit): Strike a bar: it vibrates to and fro about its rest position, an oscillatory motion, and a shorter bar vibrates with a higher frequency.
- **Sand Lissajous Pendulum** (exhibit): Swing the pendulum two ways at once: each swing repeats in equal times, and their frequency ratio sets the curve drawn in the sand.
- **Chaotic Pendulum** (exhibit): This double pendulum swings without ever quite repeating itself: unlike a single pendulum, its motion is not periodic.
- **Water Parabola** (exhibit): Spin the base: to move in circles the water needs a push toward the middle, a centripetal force, so its surface rises at the sides.
- **The Balancing Bird** (exhibit): Nudge the bird: its weighted wings put its center of gravity below its beak, so it rocks back to balance, a stable equilibrium.
- **Uphill Cone - Defying Gravity?** (exhibit): Watch the cone's middle, not its ends: its center of gravity sinks lower all the time, even while the cone seems to climb.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	10	17	19	3	**49**

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