Unit 13

Light

This unit online, with its interactives:
science-playground.com/s/10a13

On the night of 7 January 1610, in Padua, Galileo Galilei pointed a tube about a meter long, with a piece of curved glass at each end, at the planet Jupiter. Beside it he saw three small “stars” in a straight line. The next night they had moved. Within a week he had found a fourth, and understood that they were going round Jupiter: moons, the first things ever seen to go round something other than the Earth. Half a century later, in London, Robert Hooke put a thin slice of cork under another tube of lenses and saw it was made of tiny boxes. He called them **cells**, the word every biology book has used since.

Both tubes worked on the same idea: two lenses, one after the other, each doing part of the work. This unit is about lenses with numbers: how far away an image forms, how big it is, how strong a lens is, and how two lenses in a tube become a **compound microscope** or an **astronomical telescope**.

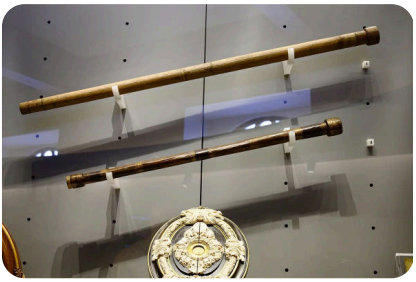


Figure 13.1 *Two of Galileo’s own telescopes, made around 1610, in the Museo Galileo, Florence. With telescopes like these he saw the moons of Jupiter, the phases of Venus and the mountains of the Moon.*
Photo: Zde, CC BY-SA 4.0

13.1 Lens

A **lens** is a piece of transparent material, usually glass or plastic, with two surfaces, at least one of them curved (part of a sphere). Light passing through it is refracted at both surfaces, and because the surfaces are curved, different parts of a beam are bent by different amounts: the rays come together, or spread apart.

Types of lens

Lenses are of two kinds, told apart by their middle.

- A **convex lens** is **thicker in the middle** than at its edges. A convex lens of glass in air bends parallel rays **together**, to meet at a point, so it is also called a **converging lens**.
- A **concave lens** is **thinner in the middle** than at its edges. It bends parallel rays **apart**, so that they seem to spread from a point, and is also called a **diverging lens**.

Each kind comes in three shapes, named by its two faces (Figure 13.2). The face named first is the one with the weaker curve; “plano” means flat.

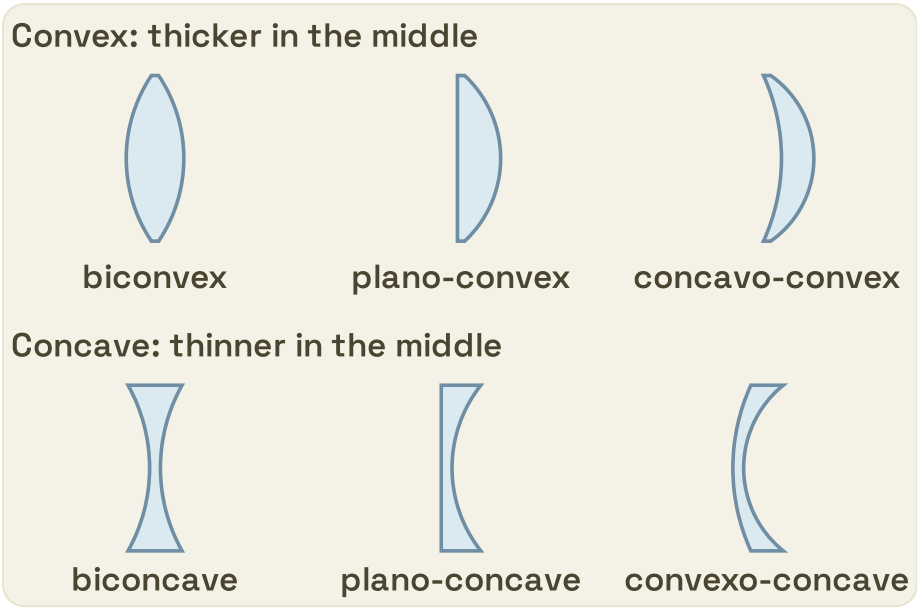


Figure 13.2 *The six shapes of lens, each cut through the middle. Whatever its faces, a lens thicker in the middle converges light, and one thinner in the middle diverges it.*

Convex (converging) lenses	Concave (diverging) lenses
Biconvex: both faces bulge outwards. A hand lens.	**Biconcave:** both faces curve inwards.
Plano-convex: one face flat, one bulging. The condenser of a projector or a microscope.	**Plano-concave:** one face flat, one curving inwards.
Concavo-convex (a converging meniscus): curved like a crescent, the outer face more curved. Spectacles for long sight.	**Convexo-concave** (a diverging meniscus): crescent-shaped, the inner face more curved. Spectacles for short sight.

Spectacle lenses are nearly always crescent-shaped (menisci), curved the same way as the eye, so that the view stays clear when the eye turns to look through the side of the lens.

The words used with lenses

Grade 10 Science explains these with drawings; here they are in one place, since every formula below uses them.

- The **optical center** (O) is the middle point of the lens. A ray through it goes straight on.
- The **principal axis** is the line through the optical center and the centers of curvature of the two faces.
- The **principal focus** (F) of a convex lens is the point on the axis where rays coming in parallel to the axis **meet** after the lens. For a concave lens, it is the point from which they **seem to spread** after the lens. A lens has a focus on each side, F_1 on the side the light comes from and F_2 on the other, at equal distances.
- The **focal length** (f) is the distance from the optical center to the principal focus.
- A **real image** is formed where rays really meet; it can be caught on a screen. A **virtual image** is where rays only seem to come from; it can only be seen by looking into the lens.

The three rays used to find an image are: a ray **parallel to the axis**, which passes through F_2 after a convex lens (or seems to come from F_1 after a concave lens); a ray **through the optical center**, which goes straight on; and a ray **through F_1** (for a concave lens, heading for F_2), which leaves parallel to the axis. Any two of them find the image.

Check 1

A lens is thinner at its middle than at its edges, with one face flat. What is it?

a a plano-convex lens, which converges light

b a plano-concave lens, which diverges light

c a concavo-convex lens, which converges light

d it depends on which face the light meets first

Power of a lens

A thick, strongly curved convex lens brings parallel light to a focus close to itself; a thin, gently curved one focuses it far away. The shorter the focal length, the more strongly the lens bends light. This bending ability is measured by the lens's power.

The **power of a lens** is the reciprocal of its focal length, measured in meters:

$$P = \frac{1}{f\ (\text{in m})}$$

Its SI unit is the **diopter** (D): a lens of focal length 1 m has a power of 1 D. A convex lens has a positive focal length and a **positive** power; a concave lens has a negative focal length and a **negative** power. That is why an eye doctor's prescription reads like "+2.5 D" (a converging lens, for long sight) or "−1.75 D" (a diverging lens, for short sight).

CAREFUL

Meters, not centimeters

Change the focal length into meters before taking its reciprocal. A lens of $f = 20$ cm has $P = 1 \div 0.20\ \text{m} = +5$ D, not $1 \div 20 = 0.05$ D. Going the other way, $f = 1 \div P$ comes out in meters: a +4 D lens has $f = 0.25\ \text{m} = 25$ cm.

Lenses in contact

Put two thin lenses together, touching, and they act as one lens whose power is the **sum** of their powers:

$$P = P_1 + P_2$$

This is why an optician testing your eyes slips one trial lens in front of another until you can read the chart: +1.00 D and +0.50 D together are +1.50 D. The eye works much the same way: the cornea (about +43 D) and the eye lens (about +20 D when relaxed) together give about +60 D (a little less than their sum, because they are a few millimeters apart, not touching), strong enough to focus light on the retina about 2 cm behind them.

WORKED EXAMPLE

Power from focal length, and back

a. Find the power of a convex lens of focal length 40 cm. b. A concave lens has a power of -2.5 D. Find its focal length.

Answer

a. $f = +40 \text{ cm} = +0.40 \text{ m}$

$$P = \frac{1}{f} = \frac{1}{0.40} = +2.5 \text{ D}$$

b.

$$f = \frac{1}{P} = \frac{1}{-2.5} = -0.40 \text{ m} = -40 \text{ cm}$$

a. **+2.5 D. b. −40 cm**: a diverging lens with its focus 40 cm in front of it.

WORKED EXAMPLE

Two lenses together

A convex lens of focal length 20 cm is placed in contact with a concave lens of focal length 50 cm. Find the power and focal length of the combination. Is it converging or diverging?

Answer

$P_1 = \frac{1}{+0.20} = +5 \text{ D}$, $P_2 = \frac{1}{-0.50} = -2 \text{ D}$

$$P = P_1 + P_2 = +5 + (-2) = +3 \text{ D}$$

$$f = \frac{1}{P} = \frac{1}{3} = 0.333 \text{ m} = 33.3 \text{ cm}$$

+3 D, focal length +33.3 cm: converging, because the convex lens is the stronger of the two.

Check 2

Which lens bends light most strongly?

a $+2$ D
b -1 D
c -8 D
d $+5$ D

Magnification

Look at a word through a hand lens held close to the page and it looks bigger; hold the lens at arm's length to look at a window across the room, and the window's image is small and upside down. How much bigger or smaller an image is than its object is given by the magnification.

The **magnification** (m) produced by a lens is the ratio of the height of the image to the height of the object:

$$m = \frac{\text{height of image}}{\text{height of object}} = \frac{h_i}{h_o}$$

It has no unit. An image with $m = 3$ is three times as tall as the object; $m = 1$, the same size; $m = 0.5$, half as tall.

Look again at Figure 13.3 below. The ray through the optical center goes straight, so the object and the image stand at the ends of two similar triangles that meet at O. Their heights are in the same ratio as their distances from the lens:

$$m = \frac{h_i}{h_o} = \frac{v}{u}$$

where u is the object's distance from the lens and v the image's. So the image is as many times bigger than the object as it is farther from the lens. Once v has a sign (in the lens formula, below), so does m: it comes out **positive for a real, inverted image** and **negative for a virtual, upright one**, and its size says how many times taller the image is.

WORKED EXAMPLE

How big is the image?

An object 2 cm tall stands 15 cm from a convex lens, and its sharp image is caught on a screen 45 cm from the lens on the other side. Find the magnification and the height of the image.

Answer

$u = 15$ cm, $v = 45$ cm, $h_o = 2$ cm

$$m = \frac{v}{u} = \frac{45}{15} = 3 \qquad h_i = m \times h_o = 3 \times 2 = 6 \text{ cm}$$

Magnification 3; the image is 6 cm tall (real, so upside down).

The lens formula

Where will the image form? Ray diagrams answer this on paper. One short formula answers it with a calculator, for every position of the object and both kinds of lens.

The sign convention. In the formula, distances are measured from the optical center, and given a sign:

- the distances of **real** objects and **real** images are taken as **positive**;
- the distances of **virtual** images are taken as **negative**;

- the focal length of a **convex** lens is **positive**, and of a **concave** lens **negative**.

This is the "real is positive" convention, the one Nepal's Grade 11 Physics book uses, and this series too.

The formula. The object distance u, the image distance v and the focal length f of a thin lens are related by the **lens formula**:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

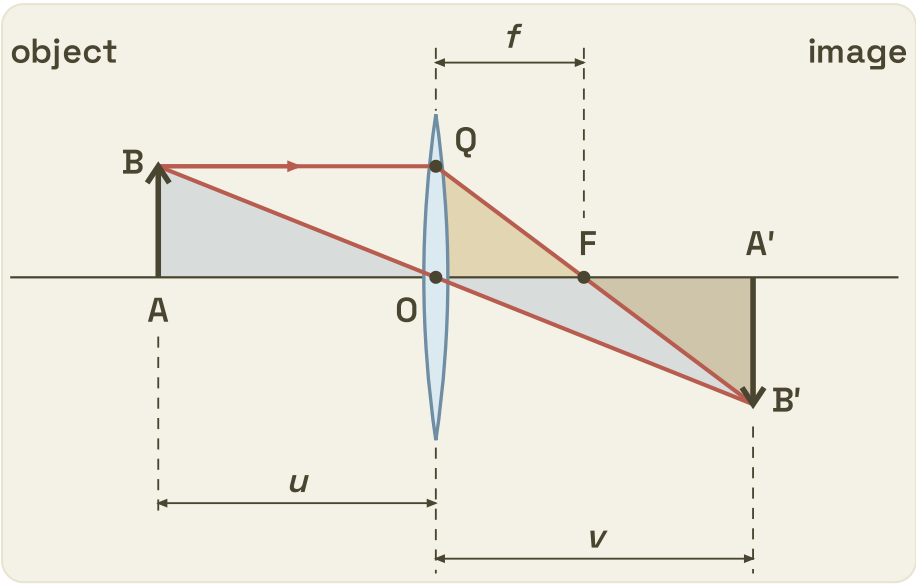


Figure 13.3 *An object AB and its real image A′B′ formed by a convex lens, drawn to scale. The ray from B parallel to the axis meets the lens at Q and passes through the focus F; the ray from B through O goes straight on. Blue: triangles ABO and A′B′O. Amber: triangles OQF and A′B′F.*

Where it comes from. In Figure 13.3, the object AB stands at a distance $u = \mathrm{OA}$ from a convex lens of focal length $f = \mathrm{OF}$, and its real image A′B′ forms at $v = \mathrm{OA'}$.

1. Triangles ABO and A′B′O (blue) are similar: their angles at O are vertically opposite, and both have a right angle on the axis. So $\dfrac{A'B'}{AB} = \dfrac{OA'}{OA} = \dfrac{v}{u}$.
2. Triangles OQF and A′B′F (amber) are similar too: their angles at F are vertically opposite, and both have a right angle on the axis. So $\dfrac{A'B'}{OQ} = \dfrac{A'F}{OF} = \dfrac{v-f}{f}$.
3. The ray from B reaches the lens parallel to the axis, so OQ = AB. The two left-hand sides are equal, so the right-hand sides are too:

$$\frac{v}{u} = \frac{v - f}{f} \quad \Rightarrow \quad vf = uv - uf \quad \Rightarrow \quad uv = vf + uf$$

Dividing every term by *uvf*:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

The same formula, with the signs above, holds for virtual images and for concave lenses as well.

> CAREFUL
> **Another way of writing it**
>
> Some books measure every distance as a coordinate from the lens, with distances against the light's direction negative (the "new Cartesian" convention). There the object distance is always negative and the formula is written $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, with $m = \frac{v}{u}$ negative for an inverted image (the opposite of the sign it has here). The two ways give exactly the same answers when each is used with its own signs; never mix a formula from one with the signs of the other. Use the one your teacher uses, and say which you are using in an exam answer. Every example and answer in this unit uses $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$.

Reading the answer. Work out $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$, then:

- v positive: a **real** image on the far side of the lens, **inverted**, which can be caught on a screen;
- v negative: a **virtual** image on the same side as the object, **upright**, seen only by looking through the lens;
- $\frac{1}{v} = 0$ (the object exactly at F): the rays leave parallel, and the image is at infinity.

Then $m = \frac{v}{u}$ carries the same sign as v: positive for a real, inverted image, negative for a virtual, upright one.

Worked examples

WORKED EXAMPLE

A real image

An object is placed 30 cm from a convex lens of focal length 20 cm. Find the position and nature of the image, and the magnification.

Answer

$u = 30$ cm, $f = +20$ cm (convex)

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{20} - \frac{1}{30} = \frac{3-2}{60} = \frac{1}{60} \qquad v = 60 \text{ cm}$$

$$m = \frac{v}{u} = \frac{60}{30} = 2$$

The image is 60 cm behind the lens: real, inverted and twice as tall as the object. It matches the table in Grade 10 Science: the object is between F and 2F, so the image is beyond 2F, magnified, as in a projector.

WORKED EXAMPLE

A magnifying glass

A stamp is held 6 cm from a convex lens of focal length 10 cm. Where is its image, and what is it like?

Answer

$u = 6$ cm, $f = +10$ cm

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{6} = \frac{3-5}{30} = -\frac{2}{30} \qquad v = -15 \text{ cm}$$

$$m = \frac{v}{u} = \frac{-15}{6} = -2.5$$

The image is 15 cm from the lens on the same side as the stamp (v is negative): virtual, upright (m is negative), and 2.5 times as big. You see it by looking through the lens, as with a magnifying glass.

WORKED EXAMPLE

A concave lens

An object is placed 30 cm from a concave lens of focal length 15 cm. Find the position, nature and magnification of the image.

Answer

$u = 30$ cm, $f = -15$ cm (concave)

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{15} - \frac{1}{30} = \frac{-2-1}{30} = -\frac{3}{30} \qquad v = -10 \text{ cm}$$

$$m = \frac{v}{u} = \frac{-10}{30} = -0.33$$

The image is 10 cm from the lens on the object's side: virtual, upright and one-third the size. A concave lens always gives a virtual, upright, diminished image, whatever u is, because $\frac{1}{v}$ is then the sum of two negative numbers.

WORKED EXAMPLE

Finding the focal length

A candle 60 cm from a lens gives a sharp image on a wall 30 cm behind the lens. Find the focal length and the power of the lens. What kind of lens is it?

Answer

$u = 60$ cm, $v = +30$ cm (real: caught on the wall)

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{60} + \frac{1}{30} = \frac{1+2}{60} = \frac{1}{20} \qquad f = 20 \text{ cm} = 0.20 \text{ m}$$

$$P = \frac{1}{0.20} = +5 \text{ D}$$

Focal length +20 cm, power +5 D: a convex lens. Only a converging lens can form a real image on a wall.

WORKED EXAMPLE

From the magnification back to the distances

A lens of power +4 D forms a real image twice as tall as the object. Find the object and image distances.

Answer

$f = \frac{1}{P} = \frac{1}{4}$ m $= 25$ cm. Real image, $m = 2$, so $v = 2u$.

$$\frac{1}{25} = \frac{1}{u} + \frac{1}{2u} = \frac{2+1}{2u} = \frac{3}{2u} \qquad 2u = 75,\ u = 37.5 \text{ cm}$$

$$v = 2u = 75 \text{ cm}$$

The object is 37.5 cm from the lens and the image 75 cm on the other side. Check: $\frac{1}{37.5} + \frac{1}{75} = \frac{2+1}{75} = \frac{1}{25}$.

WORKED EXAMPLE

A projector

A projector's lens has a focal length of 10 cm. The screen is 2.0 m from the lens. How far from the lens must the slide be, and how many times is its picture enlarged?

Answer

$f = 10$ cm, $v = 200$ cm

$$\frac{1}{u} = \frac{1}{f} - \frac{1}{v} = \frac{1}{10} - \frac{1}{200} = \frac{20-1}{200} = \frac{19}{200} \qquad u = 10.5 \text{ cm}$$

$$m = \frac{v}{u} = \frac{200}{10.53} = 19$$

The slide must be about 10.5 cm from the lens, just beyond its focus, and the picture is 19 times as large. The slide is put in upside down, so that its inverted image is the right way up.

YOUR TURN

The image at 2F

An object is 24 cm from a convex lens of focal length 12 cm. Find the image distance and the magnification.

Answer

$\frac{1}{v} = \frac{1}{12} - \frac{1}{24} = \frac{1}{24}$, so $v = 24$ cm and $m = \frac{24}{24} = 1$: a real, inverted image the same size, at 2F on the other side.

YOUR TURN

A weak spectacle lens

A pair of reading glasses has lenses of power +2 D. A newspaper is held 25 cm from one lens. Where is the image, and how big?

Answer

$f = \frac{1}{2}$ m $= 50$ cm. $\frac{1}{v} = \frac{1}{50} - \frac{1}{25} = -\frac{1}{50}$, so $v = -50$ cm: a virtual, upright image 50 cm from the lens on the paper's side, with $m = \frac{-50}{25} = -2$: twice as large. A long-sighted reader whose eye can't focus closer than about 50 cm sees the print clearly there.

Check 3

Using $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, a student finds $v = -12$ cm for a convex lens. What does the minus sign tell her?

a. she made a mistake: v can't be negative for a convex lens

b. the image is real and inverted

c. the image is virtual and upright, on the same side as the object

d. the image is 12 cm smaller than the object

Where lenses are used

- **Spectacles and contact lenses:** concave lenses (negative power) for short sight, convex lenses (positive power) for long sight and for reading in old age (see Grade 10 Science, Defects of vision).
- **Magnifying glasses**, used by watchmakers, jewelers and anyone reading small print.
- **Cameras and phone cameras**, whose lenses form real, diminished images on the sensor; **projectors**, which form real, magnified images on a screen.

- **Microscopes and telescopes**, below; **binoculars**, which are two telescopes side by side with prisms inside to turn the image the right way up.
- **Peepholes** in doors (a concave lens gives a small, upright, wide view of the doorstep), and the **condenser lenses** of projectors and microscopes.

DID YOU KNOW?

The lens in your phone

A phone's main camera is about as thick as a pencil, yet its lens is not one piece of glass but five to eight thin plastic lenses stacked together, some of them convex and some concave, with oddly curved faces. One lens alone bends the colors by slightly different amounts and blurs the edges of the picture; a stack of lenses of different shapes and powers cancels most of those faults. Together they act like one convex lens of focal length about 5 to 7 mm: over +140 D.

13.2 Optical instruments

An **optical instrument** is a device of lenses (or mirrors) that helps the eye see what it can't see well on its own: things too small, or too far away. Two are named in the course: the compound microscope and the astronomical telescope. Both are built from two convex lenses, and in both the lens at the eye, the eyepiece, is used as a magnifying glass. So that is where to start.

What a magnifying glass does

To see the details of something small, you bring it close to your eye, because nearer things look bigger. But the eye can't focus on anything closer than about **25 cm** (the **least distance of distinct vision**, D, for a young adult; see Grade 10 Science, Far point and near point). Nearer than that, the object only gets blurred.

A convex lens held close to the eye solves this. With the object just inside its focus, it forms a virtual, upright, magnified image farther away, which the eye can focus on comfortably, as in the stamp example above. Used like this, a convex lens is called a **magnifying glass** or **simple microscope**. With the image formed at 25 cm, it makes the object look

$$M = 1 + \frac{D}{f}$$

times as large as it looks to the eye alone at 25 cm (with the eye close to the lens). A hand lens of focal length 5 cm gives $1 + \frac{25}{5} = 6$ times. With the object exactly at the

focus, the image is at infinity, the eye looks at it relaxed, and the lens magnifies a little less, $\frac{D}{f}$ times: 5 times for this lens. The numbers marked on microscope eyepieces are worked out this way: a 10× eyepiece has $f = 2.5$ cm. To magnify more, the lens must have a shorter focal length, and so be smaller and more strongly curved. A single lens of focal length 1 cm would magnify 26 times, but it would be a tiny bead, hard to look through. Two lenses in a tube do far better.

> A STORY FROM SCIENCE
> **Two ways to see a cell**
>
> Antonie van Leeuwenhoek, a cloth merchant in Delft in the Netherlands, made the best microscopes of the 1600s from a **single** tiny bead of glass, a few millimeters across, held in a brass plate close to the eye. His best magnified about 270 times. Through them, in 1676, he became the first person to see bacteria, which he called "little animals". In London, Robert Hooke used a **compound** microscope, with more than one lens, easier to look through but, with the glass of the day, blurrier. In 1677 the Royal Society in London, where Hooke worked, confirmed Leeuwenhoek's "little animals". It was only in the 1800s, with better glass and lens design, that the compound microscope left the single lens far behind; and it is the compound microscope that is in every school laboratory today.

Compound microscope

Function. A **compound microscope** is an optical instrument used to see very small objects, such as cells, bacteria and the parts of a leaf, magnified many times, by means of two convex lenses: one forms a real, magnified image of the object, and the other magnifies that image again.

Basic structure. A compound microscope has:

- the **objective lens**, a convex lens of **very short focal length** (a few millimeters) and small size, close to the object. Most microscopes have three or four objectives (often marked 4×, 10×, 40× and 100×) on a turning **nosepiece**, to change the magnification;
- the **eyepiece** (or ocular), a convex lens of **short focal length** (a few centimeters, but longer than the objective's), and larger, at the top of the tube, where you put your eye. A common one is marked 10×;
- the **body tube**, a metal tube that holds the two lenses at a fixed distance apart (about 16 cm in many school microscopes);

- the **stage**, a flat plate with a hole in the middle and clips, on which the **slide** with the object rests;
- a **mirror** or a lamp, with a **condenser** lens, below the stage, which sends light up through the object (most specimens are thin and lit from below);
- the **coarse** and **fine adjustment** knobs, which move the tube (or the stage) up and down to bring the image into focus;
- the **arm** and the heavy **base**, which hold it all steady.



Figure 13.4 *A laboratory compound microscope (this one has two eyepieces, one for each eye). 1. Eyepieces. 2. Revolving nosepiece. 3. Objectives. 4. Coarse adjustment. 5. Fine adjustment. 6. Stage. 7. Lamp. 8. Condenser. 9. Mechanical stage, which holds the slide and moves it.*
Photo: GcG(jawp), Public domain

How it forms an image. The object is placed just **beyond the focus** of the objective. The objective therefore forms a **real, inverted and magnified** image of it inside the tube (the case of an object between F and 2F). This first image falls **just inside the focus** of the eyepiece. The eyepiece then acts as a magnifying glass on it, forming a **virtual, magnified** final image, usually at the least distance of distinct vision, 25 cm from the eye, or for relaxed viewing, very far away. The final image is **inverted** compared with the object: move a slide to the left and its image moves to the right.

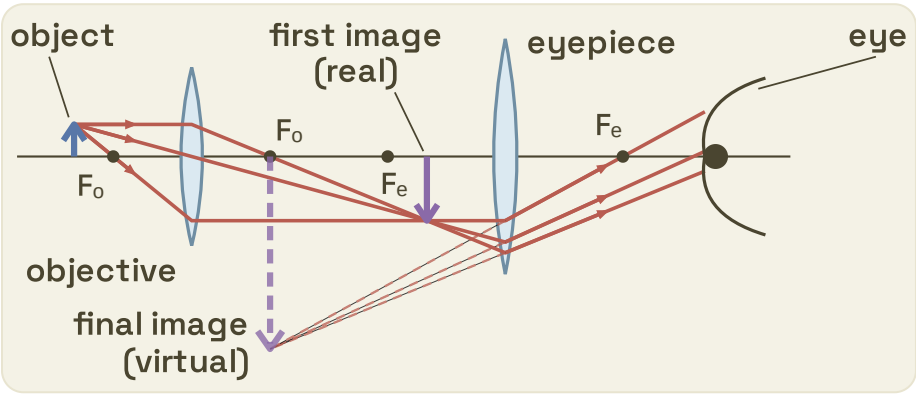


Figure 13.5 *The ray diagram of a compound microscope, drawn to one scale. The object stands beyond F_o, the objective's focus (between F_o and $2F_o$). The objective forms a real, inverted, magnified first image inside F_e, the eyepiece's focus. The eyepiece, used as a magnifying glass, sends the rays on spreading out, and the eye sees them coming from a large, virtual final image (dashed). Here the objective magnifies 2 times and the eyepiece 3 times: 6 times in all. A real microscope's lenses are much stronger.*

Magnifying power. Each lens magnifies in turn, so the whole microscope magnifies by the product (for instruments, magnifications are given as plain sizes, without signs):

$$M = m_o \times m_e$$

where $m_o = \frac{v_o}{u_o}$ is the magnification by the objective, and m_e the magnification by the eyepiece, which is $1 + \frac{D}{f_e}$ when the final image is at the least distance of distinct vision. That is why the magnification of a school microscope is found by multiplying the numbers on its lenses: a 10× eyepiece with a 40× objective magnifies 400 times. (Strictly, what a microscope or a telescope increases is the **angle** the object fills at the eye, and its **magnifying power** is the ratio of two angles; for a microscope with its image at 25 cm, it works out to the same number, $m_o \times m_e$.)

WORKED EXAMPLE

The magnification of a school microscope

A student's microscope has a 10× eyepiece and objectives of 4×, 10× and 40×. a. What magnifications can it give? b. A cheek cell is about 0.06 mm across. How big does it look with the 40× objective?

Answer

a. $M = m_o \times m_e$: $4 \times 10 = \mathbf{40\times}$, $10 \times 10 = \mathbf{100\times}$ and $40 \times 10 = \mathbf{400\times}$.

b. At 400×, the cell looks as big as an object $0.06 \times 400 = \mathbf{24\ mm}$ across would look at 25 cm from the eye: about the size of a coin.

WORKED EXAMPLE

Working out a microscope

In a compound microscope, the objective has a focal length of 1.0 cm and the eyepiece 5.0 cm. An object is placed 1.2 cm from the objective, and the final image is formed 25 cm from the eyepiece. Find a. the position of the first image, b. the magnification by each lens and the magnifying power, c. the distance between the lenses.

Answer

a. Objective: $u_o = 1.2$ cm, $f_o = 1.0$ cm

$$\frac{1}{v_o} = \frac{1}{1.0} - \frac{1}{1.2} = \frac{1.2 - 1.0}{1.2} = \frac{0.2}{1.2} \qquad v_o = 6.0 \text{ cm}$$

The first image is real, 6.0 cm behind the objective.

b. $m_o = \frac{v_o}{u_o} = \frac{6.0}{1.2} = 5$. The eyepiece forms a virtual final image at 25 cm, so $v_e = -25$ cm:

$$\frac{1}{u_e} = \frac{1}{f_e} - \frac{1}{v_e} = \frac{1}{5} + \frac{1}{25} = \frac{6}{25} \qquad u_e = 4.17 \text{ cm}$$

$m_e = \frac{25}{4.17} = 6$ in size (the same as $1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6$).

$$M = m_o \times m_e = 5 \times 6 = 30$$

c. The lenses are $v_o + u_e = 6.0 + 4.17 =$ **10.2 cm** apart.

The first image is 6.0 cm from the objective; the objective magnifies 5 times and the eyepiece 6 times, 30 times in all; the lenses are 10.2 cm apart.

CAREFUL

What the exam diagram shows

In an exam, draw the compound microscope with the first image **inside** the eyepiece's focus and the final image **virtual, inverted and magnified**, at the least distance of distinct vision. Label both images and both lenses' foci, and put arrows on every ray. A common slip is to draw the first image beyond the eyepiece's focus: then the eyepiece would make a real image and no eye could see it magnified.

Check 4

In a compound microscope, where is the object placed?

a between the objective and its focus
b just beyond the focus of the objective
c at the focus of the eyepiece
d very far from the objective

DID YOU KNOW?

How small can a microscope see?

You might think more powerful lenses would let a microscope show anything, however small. They can't. Light is a wave, and two points closer together than about half the wavelength of light (about 0.2 thousandths of a millimeter) blur into one, however much the image is magnified. So a light microscope can show bacteria (about 1 to 5 thousandths of a millimeter) but not viruses, most of which are ten times smaller. Magnifying more than about 1,500 times only makes a bigger blur. To see viruses and molecules, scientists use electron microscopes, which use beams of electrons, whose waves are many thousands of times shorter.

Astronomical telescope

Function. An **astronomical telescope** is an optical instrument used to see distant objects, such as the Moon, planets and stars, magnified and in more detail, by means of two convex lenses: the objective forms a real image of the distant object, and the eyepiece magnifies that image.

A distant object can't be brought closer, so a telescope makes it look bigger in another way: it increases the **angle** the object fills at the eye. The Moon fills an angle of only about half a degree in the sky; through a telescope that magnifies 50 times, it fills 25 degrees, and its craters can be seen.

Basic structure. An astronomical telescope has:

- the **objective lens**, a convex lens of **long focal length** and **large size**, facing the object. Its large size collects more light, so fainter things can be seen, and shows finer detail;
- the **eyepiece**, a convex lens of **short focal length** and small size, near the eye;
- the **tube**, which holds them on the same axis, usually made of two parts, one sliding inside the other, so that the distance between the lenses can be changed to focus.

How it forms an image. Light from a distant object arrives as parallel rays. The objective brings them together in its focal plane, forming a **real, inverted and much**

diminished image of the object at its focus. The tube is adjusted so that this first image lies **at the focus of the eyepiece** (in **normal adjustment**). The eyepiece then acts as a magnifying glass: rays from each point of the first image leave it parallel again, so the eye sees a final image **at infinity** and can view it relaxed, as it looks at the sky. The final image is **inverted** compared with the object, and filling a much larger angle at the eye.

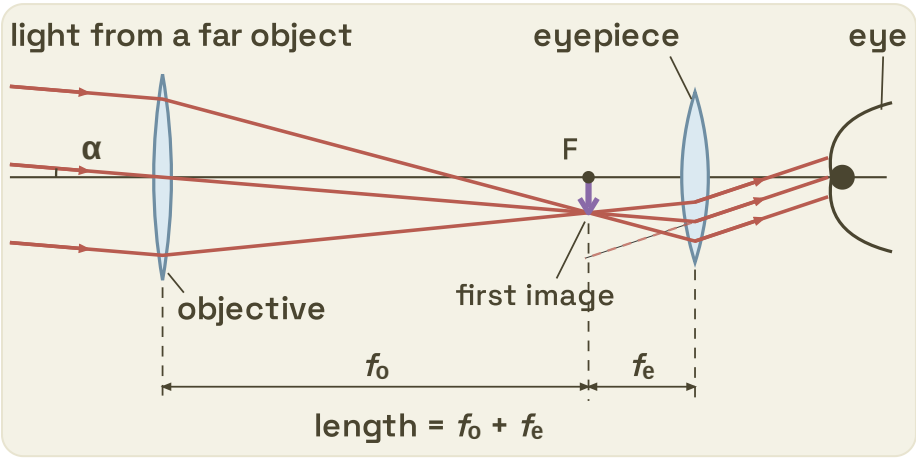


Figure 13.6 *The ray diagram of an astronomical telescope in normal adjustment, drawn to one scale. Parallel light from the top of a far object arrives at a small angle α to the axis. The objective forms a small, real, inverted first image at its focus F, which is also the focus of the eyepiece. The rays leave the eyepiece parallel again, at four times the angle, and traced back (dashed) seem to come from below the axis: the final image is inverted and at infinity. Here f_O is 4 times f_E.*

Magnifying power and length. In normal adjustment, the first image sits at both foci, so the distance between the lenses (the length of the telescope) is the sum of their focal lengths, and the magnifying power is the ratio of their focal lengths:

$$L = f_o + f_e \qquad\qquad M = \frac{f_o}{f_e}$$

A long-focus objective and a short-focus eyepiece give a large magnification, which is why astronomical telescopes are long. To magnify more, put in an eyepiece of shorter focal length.

WORKED EXAMPLE

A small telescope

An astronomical telescope has an objective of focal length 90 cm and an eyepiece of focal length 3 cm. Find its magnifying power and its length in normal adjustment.

Answer

$$M = \frac{f_o}{f_e} = \frac{90}{3} = 30 \qquad L = f_o + f_e = 90 + 3 = 93 \text{ cm}$$

It magnifies 30 times, and the lenses are 93 cm apart. The Moon, half a degree across, would fill 15 degrees.

WORKED EXAMPLE

The lenses from the length

An astronomical telescope is 66 cm long in normal adjustment and magnifies 10 times. Find the focal lengths and powers of its lenses.

Answer

$f_o + f_e = 66$ and $\frac{f_o}{f_e} = 10$, so $f_o = 10 f_e$.

$$10 f_e + f_e = 66 \qquad f_e = 6 \text{ cm} \qquad f_o = 60 \text{ cm}$$

$P_o = \frac{1}{0.60} = +1.67$ D, $P_e = \frac{1}{0.06} = +16.7$ D.

Objective 60 cm (+1.67 D), eyepiece 6 cm (+16.7 D).

CAREFUL

Upside down is fine in the sky

An astronomical telescope shows everything upside down. For the Moon and stars that doesn't matter, and adding lenses to turn the image round would cost light. A telescope for looking at things on Earth (a **terrestrial telescope**) adds a third lens between the two, to turn the image the right way up, and is longer; binoculars do the same with prisms. A **Galilean** telescope, like Galileo's own, uses a concave lens as its eyepiece, which gives an upright image in a short tube: opera glasses are built this way. The astronomical telescope, with two convex lenses, was first described by Johannes Kepler in 1611.

Compound microscope	Astronomical telescope
Used to see very small objects close by.	Used to see very distant objects.
The objective has a very short focal length and small size.	The objective has a long focal length and large size.
The objective's focal length is shorter than the eyepiece's.	The objective's focal length is longer than the eyepiece's.
The object is just beyond the objective's focus; the first image is real, inverted and **magnified**.	The object is very far away; the first image is real, inverted and **diminished**, at the objective's focus.
The final image is usually at the least distance of distinct vision (25 cm).	In normal adjustment, the final image is at infinity.
The distance between the lenses is fixed; focusing moves the whole tube towards or away from the object.	Focusing changes the distance between the lenses; in normal adjustment it is $f_o + f_e$.
$M = m_o \times m_e$	$M = \frac{f_o}{f_e}$

Check 5

To make an astronomical telescope magnify more, you should use:

a an objective of shorter focal length
b an eyepiece of shorter focal length
c two lenses of equal focal length
d a concave objective

A STORY FROM SCIENCE

Why the biggest telescopes are mirrors

The largest lens telescope ever used for research is at Yerkes Observatory in the United States, finished in 1897: its objective is 102 cm across and its tube over 18 meters long. A bigger one, 125 cm across, was shown at the Paris Exhibition of 1900, but it was hard to use, found no buyer, and was broken up for scrap. None bigger has been made since, for good reasons. A big lens can be held only by its edge, so it sags under its own weight; light has to pass through thick glass, which must be flawless all the way through; and a lens bends violet light a little more than red, so stars get colored halos. A curved **mirror** has none of these troubles: it is held from behind, light only bounces off its surface, and every color reflects the same way. So every big telescope since has used a concave mirror instead of an objective lens, from Hubble's 2.4 m mirror to the James Webb Space Telescope's 6.5 m, made of 18 gold-coated pieces. The cameras behind those mirrors often still use lenses.



Figure 13.7 *The 40-inch (102 cm) refracting telescope at Yerkes Observatory, the largest lens telescope ever used for research.*
Photo: user Kb9vrg, Public domain

IN SHORT

- A **lens** is transparent, bounded by two surfaces, at least one curved. **Convex** lenses (biconvex, plano-convex, concavo-convex) are thicker in the middle and converge light; **concave** lenses (biconcave, plano-concave, convexo-concave) are thinner in the middle and diverge it.
- **Power** $P = \frac{1}{f}$, f in meters, unit diopter (D): positive for convex, negative for concave. Lenses in contact: $P = P_1 + P_2$.
- **Magnification** $m = \frac{h_i}{h_o} = \frac{v}{u}$, no unit; with the signs of the lens formula, positive for a real image, negative for a virtual one.
- **Lens formula** $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, real distances positive, virtual negative, f positive for convex and negative for concave. v positive: real, inverted image; v negative: virtual, upright image.
- **Compound microscope:** objective of very short f, eyepiece of short f; object just beyond F_o; the first image real, inverted, magnified, inside F_e; the final image virtual, inverted, magnified. $M = m_o \times m_e$.
- **Astronomical telescope:** objective of long f and large size, eyepiece of short f; the first image real, inverted, diminished at the common focus; the final image inverted, at infinity in normal adjustment. $M = \frac{f_o}{f_e}$, length $f_o + f_e$.

Exercise

1. Choose the correct option

a. A lens that is thicker at the edges than in the middle is:

- **a** a converging lens
- **b** a diverging lens
- **c** a plano-convex lens
- **d** a concavo-convex lens

b. The SI unit of the power of a lens is:

- **a** meter
- **b** watt
- **c** diopter
- **d** centimeter^{-1}

c. The focal length of a lens of power -4 D is:

- **a** $+25$ cm
- **b** -25 cm
- **c** -4 cm
- **d** -0.4 m

d. An object placed 20 cm from a convex lens of focal length 10 cm forms an image:

- **a** 10 cm from the lens, diminished
- **b** 20 cm from the lens, the same size
- **c** 40 cm from the lens, magnified
- **d** at infinity

e. A convex lens gives a virtual image when the object is:

- **a** beyond 2F
- **b** at 2F
- **c** between F and 2F
- **d** between F and the lens

f. In a compound microscope, the first image formed by the objective is:

- **a** virtual, upright and magnified
- **b** real, inverted and magnified
- **c** real, inverted and diminished
- **d** virtual, inverted and diminished

g. The length of an astronomical telescope in normal adjustment is:

- **a** $f_o - f_e$
- **b** $f_o + f_e$
- **c** $f_o \times f_e$
- **d** $f_o \div f_e$

h. Two lenses of power $+3$ D and -1 D are placed in contact. The focal length of the combination is:

- **a** 25 cm
- **b** 50 cm
- **c** 100 cm
- **d** -50 cm

2. Give reasons

a. A convex lens is called a converging lens.

b. The power of a concave lens is negative.

c. A magnifying glass must have a short focal length.

d. The objective of an astronomical telescope is made large.

e. The final image seen through an astronomical telescope is inverted.

f. The object in a compound microscope is placed just beyond the focus of the objective, and not inside it.

3. Differentiate between

i. Convex lens and concave lens

ii. Magnification and power of a lens

iii. Compound microscope and astronomical telescope

4. Answer the following questions

a. What is a lens? Name the types of convex and concave lenses, and draw their shapes.

b. Define power of a lens and its SI unit. Why do two lenses of the same shape but different curvature have different powers?

c. State the lens formula and the sign convention used with it. Derive it for a convex lens forming a real image.

d. Describe the basic structure and function of a compound microscope, with a labeled ray diagram.

e. Describe the basic structure and function of an astronomical telescope, with a labeled ray diagram for normal adjustment.

f. Think: you have three convex lenses, of focal lengths 2 cm, 5 cm and 50 cm. Which two would you use for a microscope, and which two for a telescope? Explain.

5. Numerical problems

a. Find the power of a convex lens of focal length 25 cm, and of a concave lens of focal length 20 cm.

b. An optician prescribes spectacles of power −1.5 D. Find the focal length of the lenses and say what kind they are.

c. An object is placed 30 cm from a convex lens of focal length 15 cm. Find the position and nature of the image and the magnification.

d. An object 3 cm tall is placed 8 cm from a convex lens of focal length 12 cm. Find the position, nature and height of the image.

e. An object is placed 60 cm from a concave lens of focal length 20 cm. Find the position, nature and magnification of the image.

f. A lens forms a sharp image of a bulb on a screen 40 cm behind it when the bulb is 40 cm in front of it. Find the focal length and power of the lens.

g. A lens of power +2.5 D forms a real image four times as tall as the object. Find the object and image distances.

h. Sanskriti uses a convex lens of focal length 12 cm as a magnifying glass, and sees a virtual image of an ant three times its size. How far is the ant from the lens?

i. A convex lens of focal length 25 cm and a concave lens of focal length 10 cm are placed in contact. Find the power and focal length of the combination. Is it converging or diverging?

j. A compound microscope has an objective that magnifies 8 times and an eyepiece of focal length 5 cm, with the final image at 25 cm. Find its magnifying power.

k. In a compound microscope, the objective has a focal length of 2 cm and an object is placed 2.5 cm from it. The eyepiece has a focal length of 5 cm and the final image is formed at the least distance of distinct vision (25 cm). Find the magnifying power and the distance between the two lenses.

l. An astronomical telescope has an objective of focal length 1.2 m and an eyepiece of focal length 4 cm. Find its magnifying power and length in normal adjustment.

m. Pemba builds a telescope 55 cm long in normal adjustment that magnifies 10 times. Find the focal lengths and powers of the two lenses.

Found a mistake? Please tell us.