Unit 11

Force

This unit online, with its interactives:
science-playground.com/s/10a11

Every Dashain, villages across Nepal build a **ping**: four tall bamboo poles, crossed in pairs and lashed together high up, with a rope seat hanging from them. Watch a child on one. Pushed hard, she flies high; pushed gently, she swings low; yet the high swings and the low swings take nearly the same time. Her heavier uncle on the same ping keeps the same beat as she does. And on the bus home, as it swings round a bend on the hill road, everyone slides towards the outside of the bend, though nothing pushed them that way. Behind the bus, a truck stacked high with sacks leans on the same bend, and the driver slows right down, because a load that high could tip the whole truck over.

These are all questions about force. Why does a swing keep time, and what decides how fast it swings? What force keeps anything moving in a circle, from a stone on a string to the Moon? How strong is the Earth's pull far out in space, and how fast must a rocket go to escape it for good? And why does a tall load make a truck easy to tip? This unit answers each one, with the numbers.

11.1 Simple pendulum

Hang a key from a thread, pull it a little to one side and let go. It swings back through the lowest point, up the other side, and back again, over and over, slowly dying away. A ping, a temple bell's clapper, a chandelier in a draft and the pendulum of an old wall clock all do the same. The simplest version of all of them is the one physicists study.



Figure 11.1 *A Dashain ping. The rider and the seat swing like a pendulum hanging from where the poles are tied: you will see in 11.1 why only the length of the ropes decides how long each swing takes.*

Photo: Mithun Kunwar, CC BY 2.0

Description

A simple pendulum is a small, heavy body (the **bob**) hung from a fixed point by a light,

inextensible thread, so that it can swing freely to and fro in a vertical plane.

A perfect simple pendulum would have all its mass at one point, a thread with no mass that never stretches, and no air or friction to slow it down. No real pendulum is perfect, but a small metal ball on a long cotton thread comes very close: the ball is far heavier than the thread, so the thread's mass hardly matters, and a heavy ball loses its swing to the air only slowly.

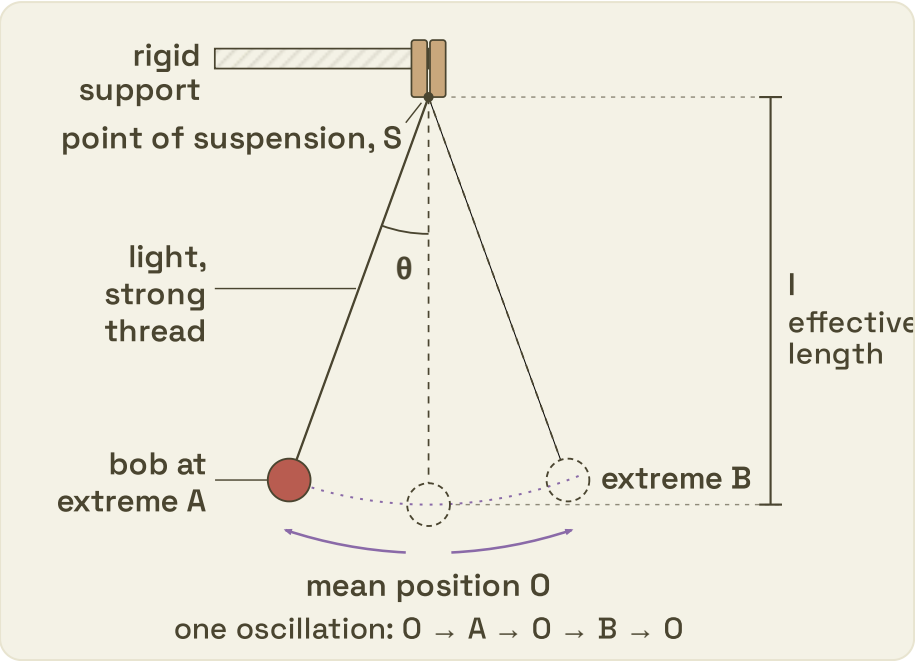


Figure 11.2 *A simple pendulum in the lab: the thread is pinched between the two halves of a split cork held in a clamp, so the point of suspension S stays fixed. The bob is shown at its extreme position A (solid), and dashed at the mean position O and the other extreme B. The effective length l is measured from S to the center of the bob.*

The words used to describe a pendulum:

- **Point of suspension (S):** the fixed point the thread hangs from. It must not move, or the pendulum loses energy to its support; that is why the thread is pinched in a split cork, not just tied round a rod.
- **Bob:** the heavy body at the end of the thread, usually a small metal sphere.
- **Effective length (l):** the distance from the point of suspension to the **center** of the bob: the length of the thread plus the radius of the bob. This is the length that decides how the pendulum swings, and the one used in every formula.

- **Mean position (O):** the position where the bob hangs at rest, straight below S. It is also called the equilibrium position.
- **Extreme positions (A and B):** the farthest points the bob reaches on either side, where it stops for an instant before turning back.
- **Amplitude:** the greatest distance of the bob from its mean position, OA or OB, measured along its path; often given as the angle θ between the thread and the vertical at the extreme position.
- **Oscillation:** one complete to-and-fro swing: from O to A, back through O to B, and back to O (or from A to B and back to A).
- **Time period (T):** the time taken for one complete oscillation, in seconds.
- **Frequency (f):** the number of complete oscillations in one second, in hertz (Hz). Since one oscillation takes T seconds, $f = \frac{1}{T}$.

CAREFUL

One oscillation is there and back

A swing from A to B is only half an oscillation; the bob must come back to A. Students who time "one swing" from one side to the other get a time half of the period. In exams, time period means the time for the whole there-and-back.

Check 1

A pendulum's thread is 95 cm long and its bob has a diameter of 4 cm. What is its effective length?

a 95 cm
b 97 cm
c 99 cm
d 91 cm

Why a pendulum swings

At the mean position, the thread pulls the bob straight up with a force equal to its weight, which pulls it straight down: the two balance and the bob can hang there at rest. Pull it aside to an angle θ and let go, and they no longer balance.

Split the bob's weight mg into two parts, as you split vectors into parts in Grade 9 Optional Science. Along the thread, the part mg cos θ is balanced by the pull of the thread (its **tension**). At right angles to the thread, along the bob's path, the part **mg sin θ** has nothing to balance it, so it pulls the bob back towards the mean position. A force like this, always pulling a body back towards its mean position, is called a **restoring force**.

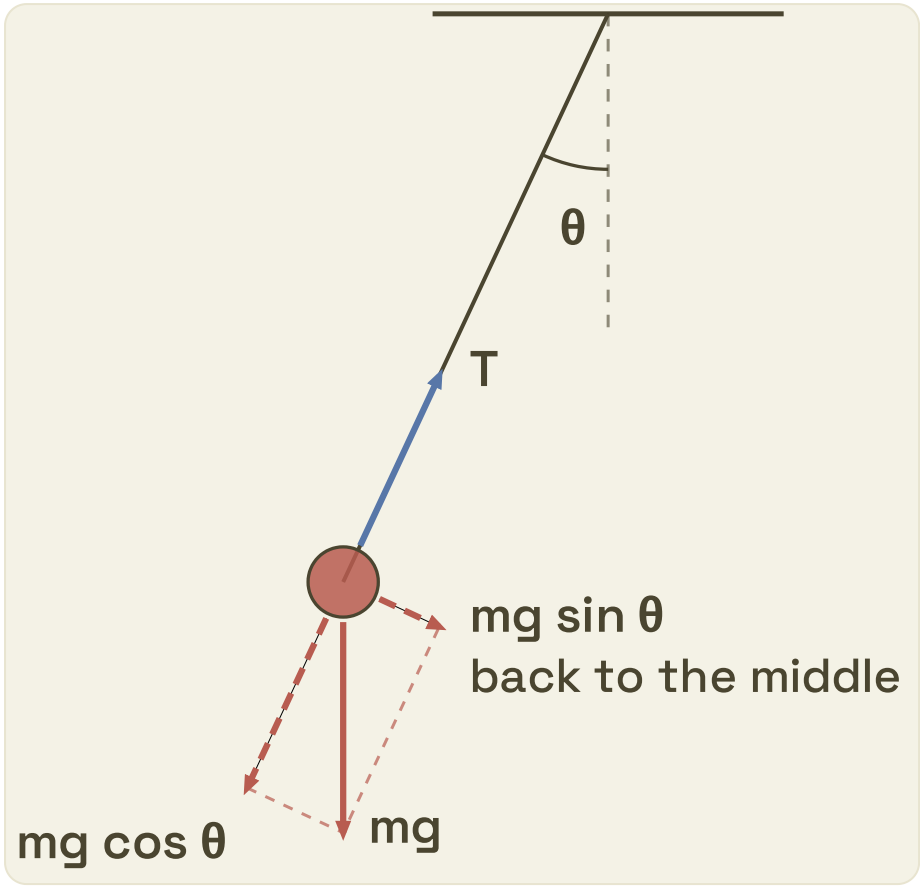


Figure 11.3 *The bob at an extreme position, the thread at θ to the vertical. Its weight mg is split into two parts: along the thread, mg cos θ, balanced by the tension T; across it, mg sin θ, which pulls the bob back along its path towards the mean position.*

The bob speeds up as it falls back, so it arrives at O moving at its fastest. There the restoring force is zero, but the bob doesn't stop: it keeps going (its inertia) and climbs up the other side, where the restoring force now pulls it back, slows it, stops it at B, and sends it back again.

In energy terms, the bob keeps trading one kind of energy for the other. At an extreme position it is highest and still for an instant: all its energy is potential energy. At the mean position it is lowest and fastest: all kinetic energy. In between, it has some of each. A little energy is lost to the air and the support at every swing, so the amplitude slowly shrinks and a real pendulum, left alone, stops in the end. A clock gives its pendulum a tiny push every swing to make up the loss.

e. The blades of a ceiling fan of radius 0.60 m turn 300 times a minute. Find the time period of the motion and the speed of the tip of a blade.

f. A stone of mass 0.5 kg tied to a 1.0 m string is whirled in a horizontal circle, making 2 rounds a second. Find its speed and the tension in the string. (Leave out gravity.)

g. A motorbike and its rider, of total mass 200 kg, go round a bend of radius 40 m. The greatest friction force the road can give the tyres is 1,800 N. What is the greatest speed at which they can take the bend without skidding?

h. The radius of the Earth is 6,400 km and g at its surface is 9.8 m/s^2. Find the gravitational field intensity (i) at a height of 3,200 km above the surface, and (ii) at a height of 6,400 km.

i. The mass of Mars is 6.4×10^{23} kg and its radius 3,400 km. Find (i) the gravitational field intensity at its surface, (ii) the gravitational potential at its surface, and (iii) the escape velocity from Mars. ($G = 6.67 \times 10^{-11}$ N m^2/kg^2)

j. The gravitational potential at the surface of a planet is -8.0×10^6 J/kg. What is the escape velocity from it? What minimum kinetic energy must a 500 kg probe have to escape from its surface?

k. A uniform meter stick of mass 120 g carries a 30 g mass at its 90 cm mark. Find the position of the center of gravity of the stick and the mass together.

l. A wooden cupboard 0.50 m deep and 1.8 m tall stands on a floor. Its center of gravity is at its middle. (i) Through what angle can it be tipped forward on its front edge before it falls? (ii) Heavy books are put on its bottom shelf, lowering its CG to 0.60 m above the floor. Through what angle can it now be tipped?

Found a mistake? Please tell us.

4. Answer the following questions

a. What is a simple pendulum? Define effective length, amplitude and time period of a simple pendulum.

b. State the laws of a simple pendulum.

c. Why does a simple pendulum, pulled aside and released, keep swinging to and fro? Explain with a diagram showing the forces on the bob at an extreme position.

d. What is periodic motion? Give four examples, and say which of them are also oscillatory.

e. What is uniform circular motion? Why is it called an accelerated motion though its speed is constant?

f. What is centripetal force? Write its formula. Describe a simple way to demonstrate it, and give three examples of its use in daily life.

g. Define gravitational field intensity. Show that it is equal to the acceleration due to gravity.

h. What is gravitational potential? What is escape velocity? Derive the relation between the escape velocity and the gravitational potential at the Earth's surface.

i. What is the center of gravity of a body? Describe how you would find the center of gravity of an irregular sheet of cardboard.

j. How is the stability of a body related to its center of gravity and its base? Explain why racing cars are made low and wide, and why loads should not be piled high on the roof of a bus.

k. Think: a pendulum clock keeps correct time in Biratnagar, on the plains. Will it run fast or slow in Kathmandu (about 1,300 m higher), and on the Moon? Explain.

5. Numerical problems

a. Find the time period and frequency of a simple pendulum of effective length 2.5 m. ($g = 9.8\ m/s^2$)

b. A simple pendulum makes 50 complete oscillations in 100 s at a place where $g = 9.8\ m/s^2$. Find its effective length.

c. In a school lab, Rajan finds that a pendulum of effective length 0.60 m makes 20 oscillations in 31.2 s. Find the value of g in the lab.

d. A pendulum has a time period of 2 s on the Earth ($g = 9.8\ m/s^2$). What would its time period be on a planet where $g = 2.45\ m/s^2$?

The time period of a simple pendulum

For small swings (an amplitude of up to about 10°), the time period of a simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where l is the effective length (in m) and g is the acceleration due to gravity (in m/s^2) at the place. Look at what is in the formula, and what is not:

- **the length l** is in it: a longer pendulum swings more slowly;
- **g** is in it: where gravity is weaker, the pendulum swings more slowly;
- **the mass of the bob is not**: a heavy bob and a light one of the same length keep the same time;
- **the amplitude is not**: a small swing and a slightly bigger one take the same time.

These four facts are stated as the **laws of the simple pendulum**:

1. **Law of isochronism:** for small amplitudes, the time period doesn't depend on the amplitude. (*Isochronous* means "taking equal times".)
2. **Law of mass:** the time period doesn't depend on the mass (or the material) of the bob.
3. **Law of length:** the time period is directly proportional to the square root of the effective length, $T \propto \sqrt{l}$.
4. **Law of acceleration:** the time period is inversely proportional to the square root of the acceleration due to gravity, $T \propto \frac{1}{\sqrt{g}}$.

Why doesn't the mass matter? A heavier bob is pulled back by a bigger restoring force, mg sin θ, but it is also harder to speed up, by exactly the same factor. The two cancel, just as they do when a heavy and a light stone fall together.

The law of isochronism holds only for small swings. For a big swing the period is a little longer than the formula gives: about 0.05 % longer at 5°, 0.4 % at 15°, 1.7 % at 30°. That is why a pendulum clock's swing is kept to a few degrees, and why, in the lab, the bob is pulled aside only a little.

WORKED EXAMPLE

The period of a 1 m pendulum

Find the time period of a simple pendulum of effective length 1.0 m in Kathmandu, where g = 9.8 m/s². How many complete oscillations does it make in a minute?

Answer

l = 1.0 m, g = 9.8 m/s²

$$T = 2\pi\sqrt{\frac{l}{g}} = 2 \times 3.14 \times \sqrt{\frac{1.0}{9.8}} = 6.28 \times 0.319 = 2.0 \text{ s}$$

In 60 s it makes 60 ÷ 2.0 = **30 oscillations**. **T = 2.0 s.**

WORKED EXAMPLE

The seconds pendulum

A **seconds pendulum** is a simple pendulum whose time period is exactly 2 s: it takes 1 s to swing each way, so it can tick the seconds of a clock. Find its length where g = 9.8 m/s².

Answer

Square both sides of $T = 2\pi\sqrt{\frac{l}{g}}$: $T^2 = 4\pi^2\frac{l}{g}$, so

$$l = \frac{gT^2}{4\pi^2} = \frac{9.8 \times 2^2}{4 \times 3.14^2} = \frac{39.2}{39.44} = 0.994 \text{ m}$$

About 0.99 m, just under a meter. (With π to more places it comes out 0.993 m; the tall clocks called grandfather clocks have a pendulum this long.)

c. In uniform circular motion, the quantity that stays constant is the body's:

a velocity b speed
c acceleration d direction of motion

d. A body of mass m moves in a circle of radius r at speed v. If both v and r are doubled, the centripetal force becomes:

a the same b twice as large
c four times as large d half as large

e. The gravitational field intensity at the Earth's surface is numerically equal to:

a the gravitational constant G b the escape velocity
c the acceleration due to gravity g d the gravitational potential

f. The escape velocity from the Earth is about:

a 7.9 km/s b 11.2 km/s
c 3×10^8 m/s d 340 m/s

g. A cone resting on its tip is in:

a stable equilibrium b unstable equilibrium
c neutral equilibrium d stable or neutral equilibrium, depending on its mass

2. Give reasons

a. A pendulum clock loses time in summer.

b. Passengers in a bus going round a bend lean outwards.

c. The roads on sharp bends of a hill highway are banked.

d. The escape velocity of a rocket is the same as that of a small stone.

e. A double-decker bus is more likely to overturn when its upper deck is full and its lower deck empty.

f. The gravitational potential at any point near the Earth is negative.

3. Differentiate between

i. Periodic motion and oscillatory motion

ii. Gravitational field intensity and gravitational potential

iii. Stable equilibrium and unstable equilibrium

iv. Centripetal force and centrifugal force

DID YOU KNOW?

Jumping over a bar your CG goes under

A high jumper using the **Fosbury flop** goes over the bar backwards, arching her back so that her head, back and legs drape round the bar. Her body is bent so much that her center of gravity, like a bangle's, lies outside her body, and it can pass **under** the bar while every part of her goes over it. She clears a higher bar for the same jump.

Check 15

A truck is loaded with sacks of cement. It is most stable if the sacks are:

a spread out low on the floor of the truck
b stacked high in the middle
c stacked high at the back
d stacked high at the front

Check 16

A ball resting on a level floor is in:

a stable equilibrium
b unstable equilibrium
c neutral equilibrium
d no equilibrium

Check 17

A body topples when:

a its center of gravity is above its base
b the vertical line through its center of gravity falls outside its base
c its base is wider than its height
d it is tilted by more than 45°

Exercise

1. Choose the correct option

a. The time period of a simple pendulum does not depend on:

a its effective length
b the acceleration due to gravity
c the mass of its bob
d the place where it swings

b. Which of these is an oscillatory motion?

a the Earth going round the Sun
b the blades of a ceiling fan
c the prongs of a vibrating tuning fork
d the hands of a clock

WORKED EXAMPLE

Measuring g with a pendulum

Kopila times 20 oscillations of a pendulum of effective length 80.0 cm and gets 36.0 s. Find g.

Answer

T = 36.0 ÷ 20 = 1.80 s; l = 80.0 cm = 0.800 m.

$$g = \frac{4\pi^2 l}{T^2} = \frac{4 \times 3.14^2 \times 0.800}{1.80^2} = \frac{31.55}{3.24} = 9.74 \text{ m/s}^2$$

g ≈ 9.7 m/s², close to the true value in Kathmandu, about 9.79 m/s². Timing 20 oscillations instead of one makes the error of starting and stopping the watch twenty times smaller in T.

WORKED EXAMPLE

Four times as long

A pendulum has a period of 1.5 s. What is its period if its length is made four times as long? And if it is made a quarter as long?

Answer

$T \propto \sqrt{l}$, so $\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}}$.

Four times as long: $T_2 = 1.5 \times \sqrt{4} = 1.5 \times 2 = \mathbf{3.0\ s}$.

A quarter as long: $T_2 = 1.5 \times \sqrt{\frac{1}{4}} = 1.5 \times \frac{1}{2} = \mathbf{0.75\ s}$.

YOUR TURN

A pendulum on the Moon

g on the Moon is 1.62 m/s². What would be the period of a 1.0 m pendulum there?

Answer

$T = 2\pi\sqrt{\frac{1.0}{1.62}} = 6.28 \times 0.786 = \mathbf{4.9\ s}$, about two and a half times as long as on the Earth. A pendulum clock taken to the Moon would run very slow.

YOUR TURN

The ping

The ropes of a ping hang 6 m from the top of the poles to the middle of the rider. Treating it as a simple pendulum, how long does each complete swing take?

Answer

$T = 2\pi\sqrt{\dfrac{6}{9.8}} = 6.28 \times 0.782 = \mathbf{4.9\ s}$. (A real ping isn't a simple pendulum, since the rider and the seat are spread out and the rider pumps, but it comes close: a tall ping really does swing slowly.)

Check 2

Two pendulums have the same length. One has a 500 g iron bob, the other a 50 g wooden bob. Both swing through small angles. Their time periods are:

a longer for the iron bob, since it is heavier

b longer for the wooden bob, since air slows it more

c the same, since the period doesn't depend on the mass of the bob

d the same only if they are pulled aside to the same angle

Check 3

To make a pendulum's time period twice as long, its effective length must be made:

a twice as long

b four times as long

c half as long

d √2 times as long



Figure 11.13 *A new coach on a tilt table in England in 1982, tilted to test its stability before it was allowed on the road. In Britain and Hong Kong a double-decker bus, loaded with weights for its passengers, must tilt to 28° without tipping over; in Hong Kong a single-decker must reach 35°.*

Photo: Rich Webb from uk, CC BY 2.0



Figure 11.14 *The Leaning Tower of Pisa, Italy. It leans 3.97° (once 5.5°, before engineers steadied it between 1993 and 2001), but the line straight down from its center of gravity still falls well inside its base, so it stands.*

Photo: Saffron Blaze, CC BY-SA 3.0

For a box of base width b with its CG at height h above the middle of its base, the line from G reaches the edge when it is tilted to an angle θ with $\tan\theta = \frac{b/2}{h}$. The lower h and the wider b, the bigger the angle it can stand.

WORKED EXAMPLE

Tipping angle of a bus

A bus has its wheels 2.0 m apart (from the middle of one tyre to the other), and when loaded with passengers its CG is 1.2 m above the road. On a road sloping sideways, at what angle would it tip over? What if heavy luggage on the roof raises its CG to 1.5 m? (Treat the bus as rigid.)

Answer

Half the base: b/2 = 1.0 m.

CG at 1.2 m: $\tan\theta = \frac{1.0}{1.2} = 0.833$, so θ = **39.8°**.

CG at 1.5 m: $\tan\theta = \frac{1.0}{1.5} = 0.667$, so θ = **33.7°**.

Raising the CG by 30 cm takes 6° off the tilt the bus can stand. (A real bus tips sooner than this: its springs and tyres squash on the downhill side and let it lean. That is why buses are tested by tilting them, below.)

Stability in daily life:

- **Loads on buses and trucks.** Heavy luggage on the roof of a bus, or sacks stacked high on a truck, raise the CG and make the vehicle easier to overturn on a bend or a sideways slope. Heavy loads should go low, in the luggage hold, and light ones on top.
- **Racing cars** are built low and wide: a low CG and a wide base.
- **A table lamp, a pressure cooker, a gas cylinder** have heavy, wide bases.
- **Carrying a load.** A porter with a doko on his back leans forward, and someone carrying a heavy bucket in one hand leans to the other side: each keeps the CG of body and load over the feet, inside the base. Standing with the feet apart widens the base; that is how you stand in a moving bus.
- **Toys that won't lie down.** A tumbler doll has a heavy, rounded base: tip it, and its very low CG is raised, so it rocks back upright.
- **A ship** carries heavy ballast low in its hull, to keep its CG low.

A STORY FROM SCIENCE

A swinging lamp in Pisa

In 1581, the story goes, a medical student named Galileo Galilei was sitting in the cathedral of Pisa, in Italy, when he noticed a lamp swinging from the high ceiling in the draft. Its swings were sometimes wide, sometimes small, but timed against the beat of his pulse they seemed to take the same time. At home he hung two pendulums of the same length, swung one widely and one gently, and found they kept time together. (The story comes from his student Viviani, written down many years later, so it may have been tidied in the telling; Galileo's own experiments with pendulums are in his books.)

Galileo saw that a pendulum could keep time, but he never built a pendulum clock. That was done on Christmas Day, 1656, by the Dutch scientist **Christiaan Huygens**. Before it, the best clocks drifted by about a quarter of an hour a day; Huygens's pendulum clock drifted by about 15 seconds. For almost three hundred years, until quartz clocks took over in the 1930s, the pendulum clock was the most accurate timekeeper in the world.

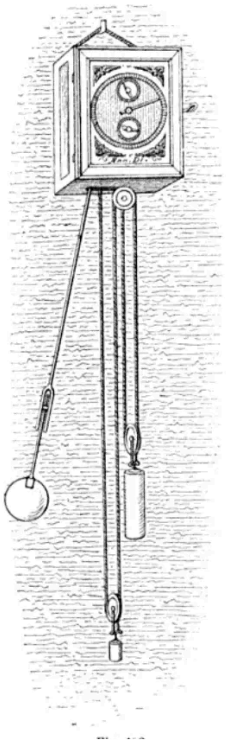


Figure 11.4 *Huygens's first pendulum clock, from his book Horologium (1658). The pendulum hangs below the clockwork, which gives it a small push at every swing.*
Photo: Unknown author, Public domain

DID YOU KNOW?

A pendulum that shows the Earth turning

In 1851 the French physicist Léon Foucault hung a 28 kg bob on a wire 67 m long from the dome of the Panthéon in Paris. Its period was about 16 seconds. Over the hours, the line of its swing slowly turned round, though nothing pushed it sideways: in fact the pendulum kept swinging in the same direction, and the floor, carried round by the turning Earth, turned beneath it. It was the first simple, direct proof of the Earth's spin, one that anyone could watch. An exact copy has swung in the Panthéon since 1995.

WORKED EXAMPLE

A meter stick with a load

A uniform meter stick of mass 100 g has a 50 g mass hung at its 10 cm mark. Where must it be supported to balance? (That point is the CG of the stick and the mass together.)

Answer

The stick's own weight acts at its CG, the 50 cm mark. Take moments about the 0 cm end; the support at x cm must carry the total weight, (100 + 50) g, with the same moment:

$$150 \times x = 100 \times 50 + 50 \times 10 = 5500 \quad \Rightarrow \quad x = \frac{5500}{150} = 36.7 \text{ cm}$$

At the 36.7 cm mark: the CG has moved from the middle towards the load. (Masses can be used instead of weights, since g cancels on both sides.)

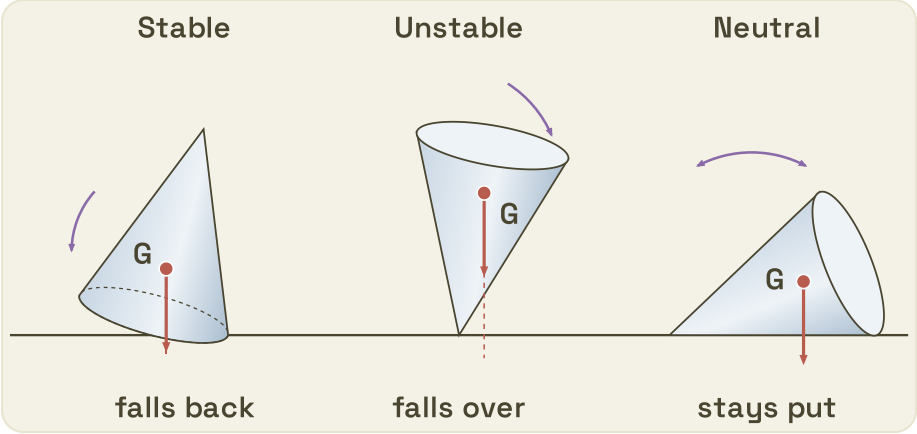


Figure 11.12 *The three kinds of equilibrium, with one solid cone. Stable: tipped a little on its base, the line straight down from G still falls inside the base, and its weight turns it back. Unstable: tipped a little on its tip, the line from G misses the tip, and its weight turns it farther over. Neutral: lying on its side, it rolls, and G stays at the same height wherever it stops.*

So a body is more stable, harder to topple, when:

1. **its center of gravity is low**, so it must be tilted farther before the line from G passes the edge of the base;
2. **its base is wide**, which also means it must be tilted farther.

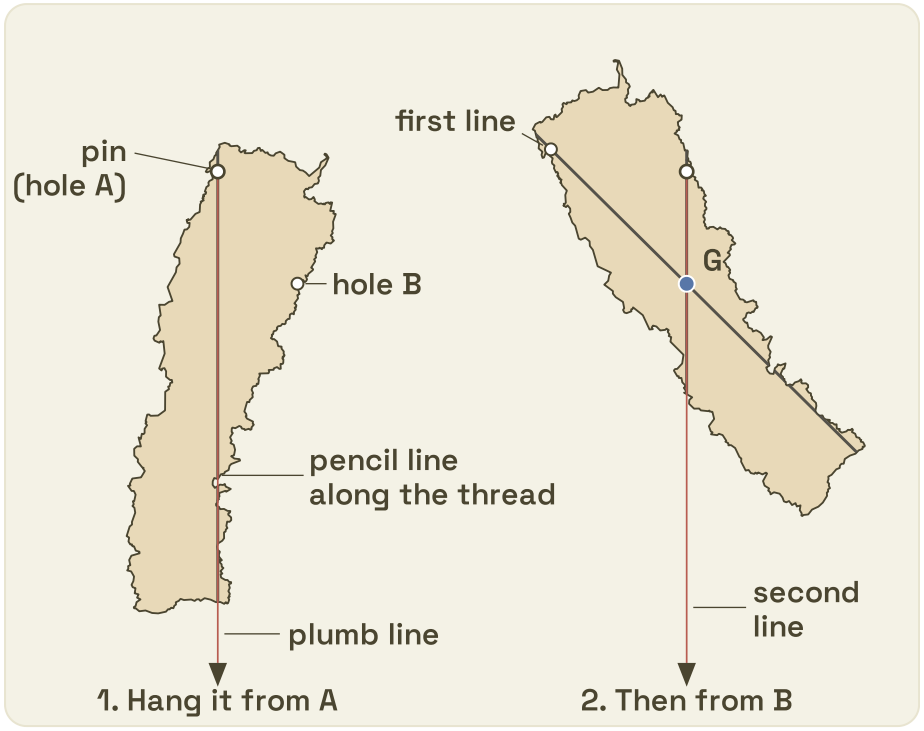


Figure 11.11 *Finding the center of gravity of a card cut in the shape of Nepal's map. 1. Hung from hole A, the card turns until G is straight below the pin; a pencil line is drawn along the plumb line. 2. Hung from hole B, the first line has turned with the card; the second line crosses it at G. (The card's G here is worked out from the map's own outline.)*

- **Stable equilibrium:** it falls back to where it was. A tilt **raises** its CG, so gravity pulls it back down. A cone standing on its base, a book lying flat, a chair on its four legs.
- **Unstable equilibrium:** it falls over, away from where it was. A tilt **lowers** its CG, so gravity pulls it farther. A cone balanced on its tip, a pencil standing on its point, a stick balanced upright on a finger.
- **Neutral equilibrium:** it stays where it is put. A tilt (here, a roll) leaves its CG at the **same height**, so gravity neither pulls it back nor farther. A ball on a level floor, a cone or a cylinder lying on its side.

When does a body topple? Draw a line straight down from its center of gravity. While that line falls **inside the base** the body stands on, its weight turns it back onto its base: it is stable. Once the body is tilted so far that the line falls **outside the base**, its weight turns it the other way, over the edge, and it topples.



Figure 11.5 *Foucault's pendulum in the Panthéon, Paris. Hour by hour, the line of its swing turns slowly round the ring.*
Photo: Rsk6400, CC BY-SA 3.0

11.2 Periodic motion

The pendulum repeats its motion over and over, each time in the same time. So does much else around you: the second hand of a clock comes back to 12 every minute, the Earth turns once a day, the Moon goes through its phases in about 29.5 days, and your heart beats about 72 times a minute.

Periodic motion is a motion that repeats itself again and again after equal intervals of time. The time taken for one repetition is the **time period** (T), and the number of repetitions in a second is the **frequency** (f), in hertz:

$$f = \frac{1}{T} \qquad \text{and} \qquad T = \frac{1}{f}$$

Periodic motion	Its time period
a seconds pendulum	2 s
the second hand of a clock	60 s
the hour hand of a clock	12 hours
the Earth spinning on its axis	1 day (24 hours)
the Earth going round the Sun	about 365¼ days
the Moon going round the Earth (new moon to new moon)	about 29.5 days
a resting heartbeat (72 beats a minute)	about 0.83 s
a tuning fork of frequency 256 Hz	1/256 s ≈ 0.0039 s
the blade of a ceiling fan at full speed	about 0.2 s

Oscillatory motion

Some periodic motions are to-and-fro about a mean position: a pendulum, a child on a ping, a mass bouncing on a spring, a guitar string, the needle of a sewing machine, a tree branch in the wind after a bird takes off. These are called **oscillatory** or **vibratory motions**. (When the to and fro is fast and small, like a guitar string's, we usually call it a vibration.)

Every oscillatory motion is periodic, but not every periodic motion is oscillatory. The hands of a clock and the Earth going round the Sun repeat themselves in equal times, but they go round and round, never to and fro about a mean position: they are periodic but not oscillatory.

A motion that doesn't repeat itself in equal times is **non-periodic**: a ball bouncing lower and lower with shorter and shorter bounces, a bus in the city traffic, a falling leaf, a footballer running about the field.

Body (uniform)	Its center of gravity
a rod, a meter stick, a pencil	its midpoint
a rectangle or a square sheet	where its diagonals cross
a triangular sheet	where its medians cross, a third of the way up from each side
a circular disc or a ring	its center
a sphere, a solid or hollow ball	its center
a cylinder	the midpoint of its axis
a solid cone	on its axis, a quarter of its height above the base

A body whose mass isn't spread evenly, like a cricket bat, a hammer or a loaded truck, has its CG nearer the heavier part: a hammer's CG is close to its head.

> CAREFUL
> **Center of gravity and center of mass**
>
> You may meet the term **center of mass**: the point where the whole mass of a body can be taken to be. For anything on the Earth, smaller than a mountain, gravity is the same on every part, and the two are the same point. (For a body as tall as a mountain, the parts nearer the Earth's center are pulled a tiny bit harder, so the center of gravity sits a tiny bit lower.)

Finding the center of gravity of simple objects

For a long, thin object like a pencil, a ruler or a meter stick, find the CG by balancing it. For a flat sheet of any shape, hang it freely: a body hanging freely from a point always comes to rest with its CG straight below that point, because only then does its weight have no moment about the point of suspension.

Relationship with stability

A book lying flat on a table is hard to knock over. Stand it up on its end and a small push tips it. Try to balance it on one corner and it falls by itself. What decides how easily a body topples is where its center of gravity is.

Tilt a body a little, and one of three things happens when you let go:

Check 13

The gravitational potential at the Earth's surface is negative because:

a gravity pushes masses away from the Earth

b the potential is zero at infinity, and gravity does work on a mass as it comes towards the Earth

c the Earth's mass is negative

d potential is a vector pointing downwards

Check 14

A planet has the same radius as the Earth but four times its mass. Its escape velocity is:

a 11.2 km/s

b 22.4 km/s

c 44.8 km/s

d 5.6 km/s

11.7 Center of gravity

Meaning

Balance a meter stick on one finger. There is just one place where it balances, the 50 cm mark, and there the stick behaves as if all its weight were pulling down at that single point. Every tiny piece of the stick is pulled down by gravity, but together their pulls act exactly as one force, the stick's weight, acting at that point.

The center of gravity (CG, often marked **G**) of a body is the point through which the whole weight of the body acts, in whatever position the body is placed.

A body supported at its center of gravity balances in any position, because its weight then passes through the support and has no moment about it (Grade 9 Optional Science: Principle of moments).

The center of gravity is not always inside the body. The CG of a ring or a bangle is at the center of the circle, in empty space; so is the CG of a boomerang, between its arms. For a uniform body of a regular shape, the CG is at its geometric center:

WORKED EXAMPLE

A tuning fork and a heart

a. A tuning fork vibrates 512 times a second. What is its time period? b. A resting heart beats 75 times a minute. What are the frequency and period of the beat?

Answer

a. f = 512 Hz, so $T = \frac{1}{f} = \frac{1}{512} = 0.00195$ s, about **2 thousandths of a second**.

b. 75 beats in 60 s: $f = \frac{75}{60} = 1.25$ Hz, and $T = \frac{1}{1.25} = \mathbf{0.8\ s}$.

DID YOU KNOW?

The pendulum in your watch

A quartz watch keeps time with a tiny tuning fork of quartz crystal, which vibrates 32,768 times a second when a small current from the battery drives it. The watch's chip counts the vibrations and moves the second hand on after every 32,768. The number isn't chosen at random: 32,768 is 2^{15}, so the chip can find each second by halving the count 15 times. A good quartz watch drifts by less than about a second a day, far better than a mechanical pocket watch, whose balance wheel can't keep time as steadily.

Check 4

Which of these is periodic but not oscillatory?

a a child on a swing

b the needle of a sewing machine

c the minute hand of a clock

d a plucked guitar string

Check 5

A pendulum makes 30 complete oscillations in 45 s. Its frequency is:

a 1.5 Hz

b 0.67 Hz

c 30 Hz

d 15 Hz

11.3 Circular motion

In Bhaktapur's Potters' Square, a potter spins a heavy wheel with a stick, and while it turns, every bit of clay on it goes round and round the same center. The tip of a ceil-

ing fan's blade, a seat on a giant wheel at a mela, the Moon round the Earth and a satellite round the Earth all do the same thing: they move along a circle.

Circular motion is the motion of a body along a circular path. When the body covers equal distances along the circle in equal times, its speed stays the same, and the motion is called **uniform circular motion**.

A body going once round a circle of radius r covers the circumference, 2πr. If one round takes the time period T, its speed is

$$v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{T}$$



Figure 11.6 *A potter at his wheel in Nepal. As the wheel turns, every point of the pot moves in a circle about the wheel's center, and points farther out move faster.*

Photo: Draavyam, CC BY-SA 4.0

Speed stays the same, velocity doesn't

In uniform circular motion the **speed** is constant, but the **velocity** is not. Velocity is a vector: it has a direction, and in circular motion the direction changes at every moment. At each point the body moves along the **tangent** to the circle there, at right angles to the radius. Go a quarter of the way round and the direction has turned through 90°.

You can see the tangent whenever something lets go of a spinning object. Water flies off a wet umbrella spun round, along the tangents, not straight outwards. Mud flies off a bicycle's tyre the same way, which is why bicycles have mudguards. Sparks from a grinding wheel are red-hot specks of metal torn off the rim, and they fly off along the tangent at the point where they leave it.

A change in velocity is an acceleration, so **a body in uniform circular motion is always accelerating**, even though its speed doesn't change. Which way? Subtract

CAREFUL

Escape velocity is not the speed rockets leave the pad at

A rocket doesn't leave the ground at 11.2 km/s; it starts slowly and keeps its engines firing, gaining height and speed for minutes, so it never needs to reach the full escape velocity at the surface. Escape velocity is the speed a body would need if it were thrown, like a stone, with no more push after it leaves. Also, don't confuse it with the speed needed to orbit just above the surface, 7.9 km/s (Grade 10 Science's Newton's cannon): escape velocity is √2 ≈ 1.41 times that.

DID YOU KNOW?

Why the Moon has no air

The molecules of a gas move very fast; at room temperature, oxygen molecules average about 0.5 km/s, and a few move much faster. On the Moon, with an escape velocity of only 2.4 km/s and days hotter than boiling water, enough of the fastest molecules escape that any air the Moon once had leaked away into space long ago. The Earth's 11.2 km/s holds on to its nitrogen and oxygen; even so, it slowly loses its lightest gases, hydrogen and helium, which move fastest.

DID YOU KNOW?

Where even light can't escape

Squeeze a body smaller and smaller, keeping its mass, and its escape velocity grows, since $v_e = \sqrt{(2GM/R)}$. Squeeze it enough and the escape velocity becomes more than the speed of light, 3×10^8 m/s. Then nothing, not even light, can get out: it is a **black hole**. For the Earth, that would mean squeezing it into a ball about 18 mm across (a radius of 9 mm), the size of a large marble. (A black hole is properly described by Einstein's theory of gravity, not Newton's; Newton's escape-velocity sum happens to give the same size.) The black hole at the center of our galaxy has about 4 million times the Sun's mass.

Three things to notice:

- **The body's mass is not in it.** A dust grain and a spacecraft need the same 11.2 km/s.
- **The direction doesn't matter** (as long as the path doesn't hit the planet): it is an energy, not a direction, that must be supplied.
- **It depends only on the planet**: its mass and radius.

Body	Escape velocity from its surface
the Moon	2.38 km/s
Mars	5.03 km/s
the Earth	11.2 km/s
Jupiter	59.5 km/s
the Sun	618 km/s

WORKED EXAMPLE

Escape velocity from the Earth

Find the escape velocity from the Earth's surface, using g = 9.8 m/s² and R = 6,400 km.

Answer

$$v_e = \sqrt{2gR} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} = \sqrt{1.254 \times 10^8} = 1.12 \times 10^4 \text{ m/s}$$

11.2 km/s, about 40,000 km/h.

WORKED EXAMPLE

Escape velocity from the Moon

The Moon's mass is 7.35×10^{22} kg and its radius 1,740 km. Find the escape velocity from its surface.

Answer

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{1.74 \times 10^6}} = \sqrt{5.64 \times 10^6} = 2.37 \times$$

About 2.4 km/s, under a quarter of the Earth's. That is why the Apollo astronauts needed only a small rocket to leave the Moon, while a huge one was needed to leave the Earth.

the velocities, as you subtracted vectors in Grade 9: take a stone going round anticlockwise, moving east (3 units) at the bottom of the circle, and a quarter turn later moving north (3 units) at the right-hand side. The change in velocity is the new velocity minus the old one.

A − B is the change in velocity: it points up and to the left, towards the center of the circle from the point halfway between the two. Take the two velocities closer and closer together, and their difference points straight at the center.



Figure 11.7 *Sparks from a grinding disc. Each spark leaves the rim along the tangent at that point and flies on in a straight line.*
Photo: Emilian Robert Vicol from Com. Balanesti, Romania, CC BY 2.0

So the acceleration of a body in uniform circular motion points **towards the center of the circle**, all the time. It is called **centripetal acceleration** (from Latin, "center-seeking"). Its size is

$$a = \frac{v^2}{r}$$

(Going Further 9–12 proves it with Pythagoras' theorem: Going round in a circle needs a pull.)

Examples of circular motion

- a potter's wheel and a grinding wheel, every point of which moves in a circle;
- the tips of the blades of a ceiling fan, and of the hands of a clock;
- a seat on a giant wheel or a merry-go-round at a mela;
- a stone whirled round on a string;
- a car or a motorbike going round a roundabout or a bend;
- the Moon round the Earth, and satellites round the Earth (their orbits are very nearly circles);
- the Earth round the Sun (very nearly a circle; the planets' orbits are ellipses, close to circles);
- the electron round the nucleus in Bohr's model of the atom Grade 9.

WORKED EXAMPLE

The rim of a potter's wheel

A potter's wheel of radius 0.40 m turns twice every second. Find the speed of a point on its rim, and of a pot being shaped 0.15 m from the center.

Answer

Two turns a second: T = 1 ÷ 2 = 0.50 s.

Rim:

$$v = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 0.40}{0.50} = 5.0 \text{ m/s}$$

Pot, 0.15 m out: $v = \dfrac{2 \times 3.14 \times 0.15}{0.50} = 1.9$ m/s.

5.0 m/s at the rim, 1.9 m/s at the pot. Every point turns once in the same time, but points farther out go round a bigger circle, so they move faster.

YOUR TURN

How fast is the Earth going?

The Earth goes round the Sun in a nearly circular orbit of radius 1.5×10^{11} m once in 365 days. Find its speed.

Answer

T = $365 \times 24 \times 60 \times 60 = 3.15 \times 10^7$ s.

$v = \dfrac{2\pi r}{T} = \dfrac{2 \times 3.14 \times 1.5 \times 10^{11}}{3.15 \times 10^7} =$ **3.0×10^4 m/s**, about 30 km/s: you travel round the Sun about 2.6 million kilometers every day.

Check 6

A stone is whirled in a circle at a steady speed. Which statement is true?

- **a** its velocity is constant, because its speed is constant
- **b** it has no acceleration, because its speed doesn't change
- **c** it is accelerating, because the direction of its velocity keeps changing
- **d** its acceleration points along the tangent

YOUR TURN

The potential at the Space Station

Find the gravitational potential at the Space Station, 400 km above the surface.

Answer

r = 6,771 km = 6.771×10^6 m. $V = -\dfrac{3.98 \times 10^{14}}{6.771 \times 10^6} =$ **-5.88×10^7 J/kg**: higher (less negative) than at the surface by 3.7×10^6 J/kg.

Escape velocity

Throw a stone up and it comes back. Throw it faster and it goes higher before it comes back. Is there a speed at which it would never come back?

There is. As the stone rises, the Earth's pull slows it down, but the pull also gets weaker as it goes, so it slows less and less. If the stone starts fast enough, its speed never quite runs out: it keeps going, slower and slower, forever. **Escape velocity** is the smallest speed with which a body must be thrown up from the surface of a planet so that it escapes the planet's gravitational field and never returns (ignoring the air). From the Earth's surface it is about **11.2 km/s** (Grade 10 Science, Future of the universe, where the rocket's graph shows it).

Its relation to gravitational potential. Escaping means climbing from the surface, where the potential energy is mV = −GMm/R, all the way to infinity, where it is zero. So the body needs to gain GMm/R of potential energy, and the only energy it has is the kinetic energy it was thrown with. Just escaping, it arrives at infinity with nothing to spare:

$$\tfrac{1}{2}mv_e^2 = \frac{GMm}{R}$$

The mass m cancels, so

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

and, since the potential at the surface is V = −GM/R,

$$v_e = \sqrt{-2V} \quad \text{or} \quad \tfrac{1}{2}v_e^2 = -V$$

The kinetic energy each kilogram needs to escape equals the depth of the potential well it starts in: the deeper the potential (the more negative V), the greater the escape velocity. For the Earth:

$$v_e = \sqrt{2 \times 6.25 \times 10^7} = \sqrt{1.25 \times 10^8} = 1.12 \times 10^4 \text{ m/s} = 11.2 \text{ km/s}$$

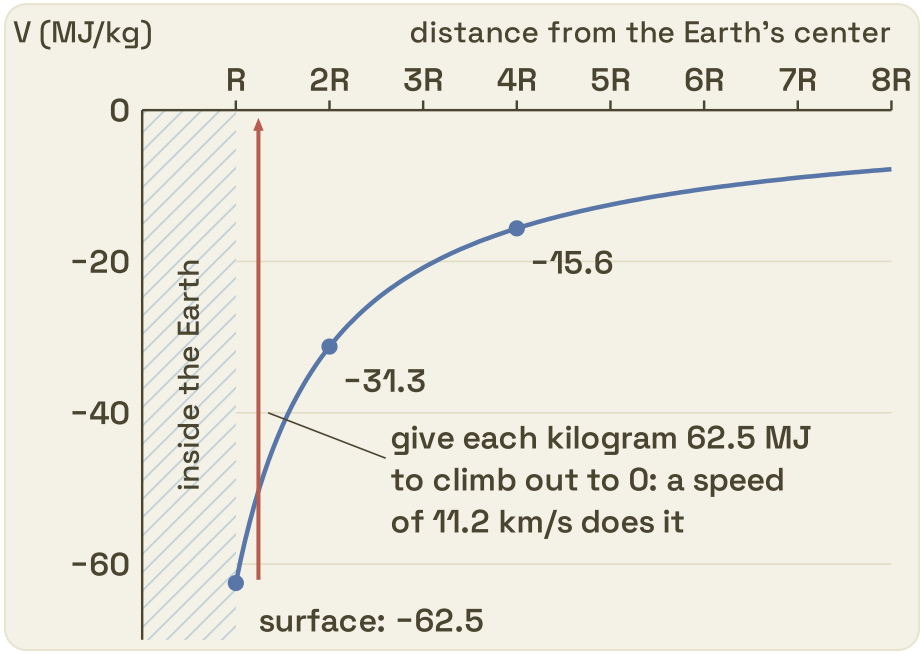


Figure 11.10 *The Earth's gravitational potential, from its surface outwards. It is −62.5 MJ/kg at the surface and rises towards zero far away: −31.3 MJ/kg at twice the radius, −15.6 at four times. A mass near the Earth sits at the bottom of a deep "well": to climb out of it for good, each kilogram must be given 62.5 MJ.*

WORKED EXAMPLE

Lifting a satellite

How much work is needed to lift a satellite of mass 1,000 kg from the Earth's surface to a point 2R from the Earth's center (one Earth radius up)? Take V = -6.25×10^7 J/kg at the surface.

Answer

At 2R, $V_2 = -\dfrac{GM}{2R} = \dfrac{-6.25 \times 10^7}{2} = -3.125 \times 10^7$ J/kg.

$W = m(V_2 - V_1) = 1000 \times \left(-3.125 \times 10^7 - (-6.25 \times 10^7)\right) = 1000 \times 3.12$

About 3.1×10^{10} J. (Using mgh with g = 9.8 would give $1000 \times 9.8 \times 6.371 \times 10^6 = 6.2 \times 10^{10}$ J, twice too much: g has fallen to a quarter by the time it gets there.)

Check 7

A wet umbrella is spun round. The drops of water fly off:

a straight outwards, along the radius

b along the tangent to the circle at the point where they leave

c inwards, towards the handle

d in a curved path round the umbrella

11.4 Centripetal force

Newton's first law says a moving body goes on in a straight line at a steady speed unless a force acts on it. A body moving in a circle doesn't go straight on: its direction keeps turning towards the center. So some force must be pulling it towards the center all the time.

Definition

Centripetal force is the force that acts on a body moving in a circle, directed towards the center of the circle, which keeps it moving in the circle.

By Newton's second law, F = ma, a body of mass m moving at speed v in a circle of radius r, whose acceleration is v^2/r towards the center, needs a force

$$F = \frac{mv^2}{r}$$

towards the center. The centripetal force is bigger:

- for a heavier body (F ∝ m),
- for a faster body, and much bigger: doubling the speed makes it four times as big (F ∝ v^2),
- for a sharper turn, a smaller circle (F ∝ 1/r).

Since $v = \dfrac{2\pi r}{T}$, the force can also be written in terms of the time for one round: $F = \dfrac{4\pi^2 mr}{T^2}$.

The centripetal force is not a new kind of force. It is the name for the job of pulling inwards, and in each case some real force does that job:

Body moving in a circle	The force that pulls it to the center
a stone whirled on a string	the tension (pull) of the string
a car or a bus going round a bend	friction between the tyres and the road
the Moon round the Earth, a satellite round the Earth	the Earth's gravitational pull
the Earth and the planets round the Sun	the Sun's gravitational pull
the electron round the nucleus (Bohr's model)	the electric attraction between them
clothes in a spinning washing machine drum	the push of the drum's wall
a rider on a giant wheel or a merry-go-round	the push of the seat and the pull of its supports

WORKED EXAMPLE

A stone on a string

A stone of mass 200 g is whirled in a horizontal circle of radius 0.50 m at 3.0 m/s. Find the pull of the string. What would it be at 6.0 m/s?

Answer

m = 200 g = 0.20 kg, r = 0.50 m, v = 3.0 m/s

$$F = \frac{mv^2}{r} = \frac{0.20 \times 3.0^2}{0.50} = \frac{0.20 \times 9.0}{0.50} = 3.6 \text{ N}$$

At 6.0 m/s, twice as fast: $F = \frac{0.20 \times 36}{0.50} = 14.4$ N.

3.6 N; four times as much, 14.4 N, at twice the speed. That is why a string whirled faster and faster snaps.

Demonstration of centripetal force

going thousands of kilometers up. We need a measure of the energy of a position in the Earth's field that works at any distance, and a zero that everyone agrees on.

The natural zero is **infinity**: a point so far away that the Earth's pull there is nothing at all. Bring a mass from there towards the Earth and gravity pulls it in; the field does work on it, and its potential energy falls. So its potential energy everywhere near the Earth is **less than zero**: negative. To take it back out to infinity, you would have to do work on it to make up the difference.

Definition

The gravitational potential at a point is the work done in bringing a unit mass (1 kg) from infinity to that point. (Equally: it is the potential energy of 1 kg placed at that point.)

Its SI unit is the **joule per kilogram (J/kg)**. Unlike field intensity, potential is a **scalar**: it is energy per kilogram, with no direction.

At a distance r from the center of the Earth (mass M), the gravitational potential is

$$V = -\frac{GM}{r}$$

The minus sign says the field, not you, does the work as the mass comes in: the potential is zero at infinity and becomes more and more negative closer to the Earth. A mass m placed at that point has potential energy U = mV = −GMm/r.

At the Earth's surface, r = R:

$$V = -\frac{GM}{R} = -\frac{3.98 \times 10^{14}}{6.371 \times 10^{6}} = -6.25 \times 10^{7} \text{ J/kg}$$

about −62.5 million joules per kilogram (−62.5 MJ/kg). Since g = GM/R², this can also be written **V = −gR** at the surface.

Potential and potential energy

Going from a point of potential V_1 to a point of potential V_2, a mass m gains potential energy $m(V_2 - V_1)$: that is the work needed to lift it.

YOUR TURN

The field of the Moon

The Moon has a mass of 7.35×10^{22} kg and a radius of 1,740 km. Find the gravitational field intensity at its surface.

Answer

$E = \dfrac{GM}{R^2} = \dfrac{6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{(1.74 \times 10^{6})^2} = \dfrac{4.90 \times 10^{12}}{3.03 \times 10^{12}} =$ **1.62 N/kg**, about a sixth of the Earth's.

DID YOU KNOW?

Weak, but enough

The Sun's gravitational field intensity at the Earth's distance is only about 0.0059 N/kg, under a thousandth of the Earth's field at its surface. Yet it is enough to keep the whole Earth, all 5.97×10^{24} kg of it, going round the Sun every year: the Earth is pulled with about 3.5×10^{22} N. A weak field acting on an enormous mass makes an enormous force.

Check 11

The SI unit of gravitational field intensity is:

a N/m^2 b N/kg
c J/kg d $N\ m^2/kg^2$

Check 12

At what distance from the Earth's center is its field intensity one ninth of its value at the surface?

a 9R b 9R above the surface
c 3R d R/3

11.6 Gravitational potential of the Earth

Concept

Lift a bag of rice onto a shelf and you do work against gravity; the work is stored in the bag as **gravitational potential energy**, given back as it falls (Going Further 9–12: Gravitational potential energy). Near the ground you work it out as mgh, from any level you choose. But g gets weaker far from the Earth, so mgh is no use for a rocket

WORKED EXAMPLE

A bus on a hill road

A loaded bus of mass 12,000 kg goes round a bend of radius 50 m at 36 km/h. What force must the road's friction give the tyres? What if the driver takes the bend at 54 km/h?

Answer

36 km/h = 36 × 1000 ÷ 3600 = 10 m/s.

$$F = \frac{mv^2}{r} = \frac{12\,000 \times 10^2}{50} = 24\,000 \text{ N}$$

54 km/h = 15 m/s: $F = \dfrac{12\,000 \times 15^2}{50} = 54\,000$ N.

24,000 N at 36 km/h, but 54,000 N, 2.25 times as much, at 54 km/h. On a wet or gravelly road the tyres may not be able to grip that hard, and the bus slides off the bend; that is why the signs before a bend say slow down.

YOUR TURN

Find the radius

A 1,000 kg car goes round a roundabout at 8 m/s, and the friction on its tyres towards the center is 3,200 N. What is the radius of its circle?

Answer

$r = \dfrac{mv^2}{F} = \dfrac{1000 \times 8^2}{3200} = \dfrac{64\,000}{3200} =$ **20 m**.

DID YOU KNOW?

The bucket that doesn't spill

Fill a bucket half full of water and swing it quickly round in a vertical circle at arm's length. At the top of the circle the bucket is upside down, yet the water stays in. At the top, gravity is pulling the water down towards the center of the circle, and if the bucket moves fast enough, gravity alone isn't enough to keep the water on such a tight circle; the bucket's bottom must push on it too, so the water presses into the bucket. The water stays in as long as gravity alone isn't enough: as long as v^2/r is at least g, so v is at least $\sqrt{(gr)}$. For an arm and bucket 0.9 m long, that is $\sqrt{(9.8 \times 0.9)} \approx 3$ m/s at the top. Swing slower, and you get wet.

CAREFUL

Centripetal, not centrifugal

On a bus going round a bend you feel thrown outwards, as if a force pushed you away from the center. Nothing pushes you that way. Your body is only trying to go on in a straight line (its inertia), and the bus turns away under you, until the side of the seat pushes you inwards and makes you go round with it. The real force on you points inwards: centripetal.

The outward "force" you feel is called the **centrifugal force** ("center-fleeing"). It is not a push or pull by anything: physicists add it only when they describe the motion from inside the turning bus, as if the bus were standing still, to make Newton's laws work there. That is why it is called a **pseudo force** (or fictitious force). It has the same size as the centripetal force, mv^2/r, and points away from the center.

In the exam: the force that keeps a body moving in a circle is always the **centripetal** force, towards the center. If a question asks about centrifugal force, write that it is a pseudo force, of size mv^2/r, directed away from the center, felt only by someone going round with the body. Some books say the centrifugal force is the reaction to the centripetal force. It isn't: the reaction to the string's pull on a whirled stone is the stone's pull on the string and your hand, which acts on your hand, not on the stone.

Daily-life applications

- **Banked roads and tracks.** On a flat road, only friction turns a vehicle, and on a wet road friction may not be enough. So the roads of hill highways, the curves of racing tracks and railway lines on a bend are **banked**: the outer edge is raised above the inner one. The road then pushes on the tyres at a slant, partly inwards, and that inward part helps provide the centripetal force.
- **A cyclist leaning into a bend.** A cyclist or a motorcyclist leans inwards on a turn. The road then pushes up and inwards on the tyres, and the inward push gives the centripetal force (the faster or sharper the turn, the more they must lean).
- **Speed limits on bends.** Since $F = mv^2/r$, a sharp bend at high speed needs a very large force. Signs before sharp bends on Nepal's hill roads ask drivers to slow down: half the speed needs only a quarter of the force.
- **Spin dryers and washing machines.** The drum spins very fast and pushes the clothes round in a circle. The water in the clothes isn't held in so firmly; it goes straight on, out through the holes in the drum, and the clothes are left almost dry.

WORKED EXAMPLE

The force from the field

At a point near the Space Station the Earth's gravitational field intensity is 8.7 N/kg. What is the gravitational force on an astronaut of mass 70 kg there?

Answer

$$F = mE = 70 \times 8.7 = 609 \text{ N}$$

About 610 N, about nine tenths of the 686 N she weighs on the ground. She floats in the station not because the field is gone, but because she and the station are both falling round the Earth together.

WORKED EXAMPLE

The field 1,000 km up

Find the Earth's gravitational field intensity 1,000 km above its surface.

Answer

$r = R + h = 6{,}371 \text{ km} + 1{,}000 \text{ km} = 7{,}371 \text{ km} = 7.371 \times 10^6 \text{ m}$.

$$E = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(7.371 \times 10^6)^2} = \frac{3.98 \times 10^{14}}{5.433 \times 10^{13}} = 7.33 \text{ N/kg}$$

7.33 N/kg.

WORKED EXAMPLE

From the surface value

The field intensity at the Earth's surface is 9.8 N/kg. Find it at a height equal to the Earth's radius above the surface, without using G or M.

Answer

At the surface, r = R; at height R, r = 2R. Since $E \propto \frac{1}{r^2}$:

$$\frac{E_2}{E_1} = \frac{R^2}{(2R)^2} = \frac{1}{4} \quad \Rightarrow \quad E_2 = \frac{9.8}{4} = 2.45 \text{ N/kg}$$

2.45 N/kg. Watch the trap: "a height equal to R" means 2R from the center, not R.

For the Earth, a mass m at a distance r from its center feels a force $F = \frac{GMm}{r^2}$ (Newton's law of gravitation, Grade 10 Science), where M is the Earth's mass. So

$$E = \frac{F}{m} = \frac{GM}{r^2}$$

This is exactly the formula for **g**, the acceleration due to gravity Grade 10. **The gravitational field intensity at a point is equal to the acceleration due to gravity there.** At the Earth's surface it is 9.8 N/kg: every kilogram is pulled down with 9.8 N, and that pull, acting alone, accelerates it at 9.8 m/s². The two units are the same thing: 1 N/kg = 1 (kg m/s²)/kg = 1 m/s².

Since E = GM/r², the field gets weaker with distance as 1/r²: at twice the distance from the center, a quarter of the strength; at three times, a ninth.

Place	Distance from the Earth's center	Field intensity
sea level	6,371 km (R)	9.81 N/kg
the top of Sagarmatha (8,849 m)	6,380 km	9.78 N/kg
the Space Station (400 km up)	6,771 km	8.69 N/kg
1,000 km up	7,371 km	7.33 N/kg
2R from the center (6,371 km up)	12,742 km	2.45 N/kg
geostationary satellites (35,786 km up)	42,157 km	0.224 N/kg
the Moon's distance	384,400 km	0.0027 N/kg

(Here and below, G = 6.67×10^{-11} N m²/kg², the Earth's mass M = 5.97×10^{24} kg and its radius R = 6,371 km, so GM = 3.98×10^{14} N m²/kg.)

- **Cream separators and centrifuges.** Milk spun very fast separates: the heavier, watery milk goes to the outside, the lighter cream stays nearer the center and is drawn off. In a hospital laboratory, a **centrifuge** spins tubes of blood so that the heavier blood cells pack at the bottom of each tube, and the clear plasma can be tested.
- **The hammer throw.** An athlete swings a heavy ball on a wire round and round, pulling it inwards with all her strength, then lets go: the ball flies off along the tangent.
- **The Moon and satellites.** The Earth's gravity is the centripetal force that keeps the Moon and every satellite on its orbit; without it, they would fly off in straight lines (Going Further 9–12: Gravity does the pulling).



Figure 11.8 *A hammer thrower spins with the ball on its wire. Her pull on the wire is the centripetal force; the moment she lets go, the ball flies off along the tangent.*
Photo: HeavyHazmatt, CC BY-SA 4.0

DID YOU KNOW?

Newton's test with the Moon

Newton asked whether the force that makes an apple fall is the same one that keeps the Moon on its orbit. The Moon goes round the Earth at a distance of 3.84×10^8 m once in 27.3 days, so its speed is 2πr/T ≈ 1,020 m/s and its centripetal acceleration is v²/r ≈ 0.0027 m/s². The Earth's gravity at that distance, worked out from g = GM/r² (as in 11.5), is also about 0.0027 m/s². They agree: the Earth's gravity does exactly the Moon's centripetal job. The Moon falls towards the Earth all the time, as the apple does, but it is moving sideways so fast that it keeps missing.

Check 8

A car goes round a bend at twice its earlier speed. The centripetal force needed is:

a the same
b twice as much
c four times as much
d half as much

Check 9

What provides the centripetal force for the Moon going round the Earth?

a the Moon's own speed
b a centrifugal force pushing it outwards
c the gravitational pull of the Earth on the Moon
d the gravitational pull of the Sun on the Moon

Check 10

Why are roads banked on sharp bends?

a to let rainwater run off the road
b so that vehicles can't go fast
c so that the road's push on the tyres helps provide the centripetal force
d to give drivers a better view round the bend

11.5 Gravitational field intensity

Introduction

A magnet pulls a pin from a distance, without touching it; the space round the magnet where it can do that is its magnetic field. Gravity works the same way. The Earth pulls the Moon, a satellite and a falling mango without touching any of them. The space round any mass in which another mass feels a gravitational force is called its **gravitational field**. Every mass has one, but only a very large mass like the Earth, the Moon or the Sun has a field strong enough to notice.

A field is drawn with **gravitational field lines**, which show the direction in which a mass placed there would be pulled. Round the Earth they point straight in towards its center from every side. Where the lines are closer together, near the surface, the field is stronger; farther out they spread apart, and the field is weaker.

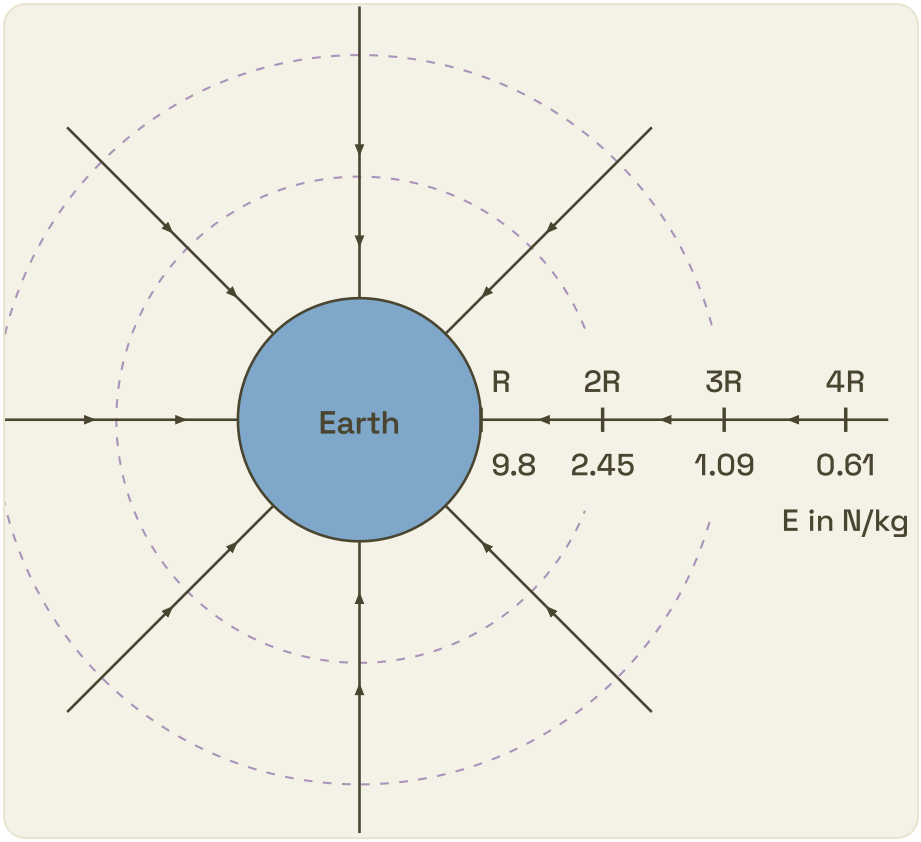


Figure 11.9 *The Earth's gravitational field. The field lines all point in towards the center. Along one of them, the field intensity at R, 2R, 3R and 4R from the center (R is the Earth's radius) is 9.8, 2.45, 1.09 and 0.61 N/kg: twice as far, a quarter as strong.*

Gravitational field intensity

How strong is a field at a point? Place a mass there and measure the force on it. A 2 kg mass feels twice the force of a 1 kg mass at the same point, so the fair measure is the force on each kilogram.

The gravitational field intensity at a point is the gravitational force acting on a unit mass (1 kg) placed at that point:

$$E = \frac{F}{m}$$

Its SI unit is the **newton per kilogram (N/kg)**. It is a vector, pointing the way a mass there would be pulled: towards the body making the field.