Unit 11

Answers: Force

Quick checks

1 b The effective length runs from the point of suspension to the center of the bob: thread + radius = 95 + 4 ÷ 2 = 97 cm. 99 cm adds the whole diameter; 91 cm takes it away.

2 c $T = 2\pi\sqrt{l/g}$ has no mass in it: the law of mass. And for small swings it doesn't matter how far each is pulled aside either (isochronism). Air slows the light bob's swing down sooner, but it barely changes its period.

3 b T ∝ √l, so doubling T needs √l to double, which means l must become $2^2 = 4$ times as long.

4 c The minute hand repeats its motion every hour, so it is periodic, but it goes round and round, never to and fro about a mean position. The other three all oscillate about a mean position.

5 b T = 45 ÷ 30 = 1.5 s, so f = 1 ÷ 1.5 = 0.67 Hz (or directly: 30 oscillations ÷ 45 s). 1.5 is the period in seconds, not the frequency.

6 c Velocity has a direction, and in circular motion the direction changes all the time, so the velocity changes: the stone accelerates, towards the center. The velocity is along the tangent; the acceleration is at right angles to it, along the radius, inwards.

7 b While it clings to the cloth, a drop is pulled round in a circle. When it lets go, nothing pulls it sideways any more, so it goes straight on in the direction it was moving at that instant: along the tangent.

8 c $F = mv^2/r$: the force grows with the square of the speed, so twice the speed needs $2^2 = 4$ times the force.

9 c The Earth's gravity pulls the Moon towards the Earth's center, which is the center of its orbit: that pull is the centripetal force. Speed isn't a force, and no outward force acts on the Moon at all.

10 c On a banked road the road pushes on the tyres at a slant, partly towards the center of the bend, so less depends on friction, and a vehicle can turn safely even on a wet road.

11 b Field intensity is force per unit mass, so newtons per kilogram, which is the same as m/s^2. J/kg is the unit of gravitational potential (next section); $N\ m^2/kg^2$ is the unit of G.

12 c E ∝ $1/r^2$, so a ninth of the field means r^2 is 9 times as big: r = 3R from the center (2R above the surface).

13 b Potential is measured from zero at infinity. Bringing a mass in from there, the attracting field does the work, so the mass ends up with less energy than at infinity: a negative potential. Potential is a scalar.

14 **b** $v_e = \sqrt{(2GM/R)} \propto \sqrt{M}$ at the same R. Four times the mass gives $\sqrt{4} = 2$ times the escape velocity: $2 \times 11.2 = 22.4$ km/s.

15 **a** Spreading the load low keeps the CG low, so the truck must be tilted farther before the line from its CG passes outside its wheels. Stacking high raises the CG and makes it easier to overturn.

16 **c** Rolled to any position, the ball's CG stays at the same height above the floor, so it neither comes back nor rolls on: neutral equilibrium.

17 **b** While the line straight down from G falls inside the base, the weight turns the body back; once it falls outside, the weight turns it over. The angle at which that happens depends on the body: 45° only for a box as wide as twice its CG's height.

1 a. **c** $T = 2\pi\sqrt{(l/g)}$: it depends on the length and on g (which changes from place to place), but not on the bob's mass.

1 b. **c** The prongs move to and fro about their mean positions. The other three are periodic, but they go round, not to and fro.

1 c. **b** Its speed stays the same. Its velocity and acceleration are vectors whose directions keep turning, so they change all the time even though their sizes don't.

1 d. **b** $F = mv^2/r$: doubling v multiplies it by 4, doubling r divides it by 2, so overall it doubles.

1 e. **c** $E = F/m = GM/R^2$, which is exactly g: 9.8 N/kg = 9.8 m/s².

1 f. **b** $v_e = \sqrt{(2gR)} \approx 11.2$ km/s. 7.9 km/s is the speed to orbit just above the surface; 3×10^8 m/s is the speed of light; 340 m/s is the speed of sound in air.

1 g. **b** The slightest tilt lowers its center of gravity, and the line from G then misses the tip, so its weight turns it farther over: unstable.

2 a. In summer the metal rod of the pendulum expands, so its effective length increases. Since $T = 2\pi\sqrt{(l/g)}$, the time period increases: each swing takes longer, the clock ticks fewer times in an hour than it should, and it runs slow (loses time).

2 b. By inertia, a passenger's body tends to go on moving in a straight line. When the bus turns, it turns away from that line, and until the seat or the side of the bus pushes the passenger round with it (the centripetal force), the passenger's body goes on straight and so seems to lean or slide outwards, away from the center of the bend.

2 c. A vehicle going round a bend needs a centripetal force towards the center. On a flat road only friction between the tyres and the road provides it, and on a wet or gravelly road friction may not be enough, so the vehicle skids outwards. When the outer edge of the road is raised, the road pushes on the tyres at a slant, partly towards the center of the bend, and this inward part helps provide the centripetal force, so vehicles can take the bend safely.

2 d. To escape, the kinetic energy given must equal the depth of the potential energy: $\frac{1}{2}mv_e^2 = GMm/R$. The mass m of the body appears on both sides and cancels, so $v_e = \sqrt{(2GM/R)}$ depends only on the planet's mass and radius, not on the mass of what is thrown.

2 e. With the passengers on the upper deck, the center of gravity of the bus is higher. A higher CG means the bus needs only a smaller tilt before the vertical line through its CG passes outside its base (the wheels), so it overturns more easily on a bend or a sideways slope.

2 f. Gravitational potential is taken as zero at infinity. When a unit mass is brought from infinity to a point near the Earth, the Earth's attraction does the work, not an outside agent, so the mass has less potential energy at that point than at infinity: its potential is less than zero, that is, negative ($V = -GM/r$).

3 i.

Periodic motion	Oscillatory motion
A motion that repeats itself after equal intervals of time.	A periodic motion to and fro about a mean position.
May be round and round, or to and fro.	Always to and fro about a mean position.
Not every periodic motion is oscillatory.	Every oscillatory motion is periodic.
Example: the Earth going round the Sun, the hands of a clock.	Example: a simple pendulum, a vibrating guitar string.

3 ii.

Gravitational field intensity	Gravitational potential
The force on a unit mass at a point.	The work done in bringing a unit mass from infinity to a point.
$E = F/m = GM/r^2$.	$V = -GM/r$.
Unit: N/kg.	Unit: J/kg.
A vector, pointing towards the mass.	A scalar.
Falls as $1/r^2$ with distance.	Rises towards zero as $1/r$ with distance.

3 iii.

Stable equilibrium	Unstable equilibrium
A small tilt raises the center of gravity.	A small tilt lowers the center of gravity.
After a small tilt, the body returns to its first position.	After a small tilt, the body falls farther away from its first position.
The vertical through G stays inside the base.	The vertical through G falls outside the base (or point of support).
Example: a cone on its base, a book lying flat.	Example: a cone on its tip, a pencil on its point.

3 iv.

Centripetal force	Centrifugal force
A real force (tension, friction, gravity…) acting on a body moving in a circle.	A pseudo force: not a push or pull by anything, but the body's inertia, its tendency to go straight on, as felt by someone going round with it.
Directed towards the center of the circle.	Felt as directed away from the center.
$F = mv^2/r$; without it the body can't move in a circle.	The same size, mv^2/r, felt by passengers in a turning bus, people on a merry-go-round.

4 a. A simple pendulum is a small, heavy bob hung from a rigid support by a light, inextensible thread so that it can swing freely in a vertical plane.

- Effective length: the distance from the point of suspension to the center of the bob (length of thread + radius of bob).
- Amplitude: the greatest displacement of the bob from its mean position, measured along its path or as the angle the thread makes with the vertical at the extreme position.
- Time period: the time taken for one complete oscillation (O to A to O to B to O).

4 b.

1. Law of isochronism: for small amplitudes, the time period doesn't depend on the amplitude.
2. Law of mass: the time period doesn't depend on the mass or material of the bob.
3. Law of length: the time period is directly proportional to the square root of the effective length, $T \propto \sqrt{l}$.
4. Law of acceleration: the time period is inversely proportional to the square root of the acceleration due to gravity, $T \propto 1/\sqrt{g}$.

Together: $T = 2\pi\sqrt{(l/g)}$.

4 c. At an extreme position the bob's weight mg acts vertically down while the thread makes an angle θ with the vertical. The weight has two parts: mg cos θ along the thread, balanced by the tension, and mg sin θ at right angles to the thread, along the bob's path, towards the mean position. This restoring force pulls the bob back, and it speeds up. At the mean position the restoring force is zero, but the bob is moving fastest, so by its inertia it carries on up the other side, where the restoring force now acts the other way, slows it, stops it and sends it back. In energy terms, potential energy at the extremes changes to kinetic energy at the mean position and back. Diagram: the bob on its thread at angle θ, with mg downward, mg cos θ along the thread, T up the thread and mg sin θ along the path towards the mean position, as in Figure 11.3.

4 d. Periodic motion is a motion that repeats itself after equal intervals of time. Examples: a simple pendulum, a vibrating guitar string, the Earth spinning on its axis, the hands of a clock (also the heartbeat, the Moon round the Earth). The pendulum and the guitar string are also oscillatory, since they move to and fro about a mean position; the Earth's spin and the clock hands are periodic but not oscillatory.

4 e. Uniform circular motion is the motion of a body along a circular path at a constant speed. Its velocity is along the tangent to the circle, and the direction of the tangent changes at every point. Since velocity has direction, the velocity keeps changing even though the speed doesn't, and a change in velocity is an acceleration. This acceleration is directed towards the center of the circle (centripetal acceleration, v^2/r).

4 f. Centripetal force is the force acting on a body moving in a circle, directed towards the center of the circle, which keeps it in the circle: $F = mv^2/r$.
Demonstration: tie a small ball to a string, pass the string through an empty pen tube and hang a few washers on its lower end. Whirl the ball in a horizontal circle above your head: the washers' weight pulls the string, and so the ball, towards the center, and the ball can be kept going round. Whirl faster and the washers rise, since a bigger force is needed; let go, and the ball flies off along the tangent.
Daily-life uses: banked roads on bends; a cyclist leaning on a turn; spin dryers and centrifuges (cream separators, blood centrifuges); the Earth's gravity keeping satellites in orbit.

4 g. Gravitational field intensity at a point is the gravitational force acting on a unit mass placed there: $E = F/m$ (unit N/kg).
A mass m at distance r from the center of the Earth (mass M) feels $F = GMm/r^2$, so $E = F/m = GM/r^2$. The same force gives the mass an acceleration g, so $F = mg$, and $g = F/m = GM/r^2$. So $E = g$: the field intensity at a point is equal to the acceleration due to gravity there (9.8 N/kg = 9.8 m/s² at the surface).

4 h. Gravitational potential at a point is the work done in bringing a unit mass from infinity to that point; at the Earth's surface $V = -GM/R$.
Escape velocity is the smallest speed with which a body must be projected from the surface so that it escapes the Earth's gravitational field and never returns.
To escape, a body of mass m must climb from potential energy $mV = -GMm/R$ to zero at infinity, a gain of GMm/R, using its kinetic energy. Just escaping: $\frac{1}{2}mv_e^2 = GMm/R$, so $v_e = \sqrt{(2GM/R)}$. Since $V = -GM/R$, $\frac{1}{2}v_e^2 = -V$, or $v_e = \sqrt{(-2V)}$. The deeper (more negative) the potential, the greater the escape velocity. Also, since $GM = gR^2$, $v_e = \sqrt{(2gR)} \approx 11.2$ km/s.

4 i. The center of gravity of a body is the point through which its whole weight acts, whatever its position.
Method: make two or three small holes near the edge of the card. Hang it freely from a pin through one hole, with a plumb line hanging from the same pin. When it is still, mark the line of the plumb line on the card and draw it. Hang the card from a second hole and draw the second line in the same way. The point where the lines cross is the center of gravity. Hanging it from a third hole checks it: the third line passes through the same point. (It works because a freely hanging body comes to rest with its CG directly below the point of suspension.)

4 j. A body is stable while the vertical line through its center of gravity falls inside its base; once a tilt takes that line outside the base, it topples. So a body is more stable when its CG is low and its base is wide, because then it must be tilted farther before the line passes the edge ($\tan\theta = (b/2)/h$).
Racing cars are built low, which keeps their CG close to the road, and wide, which gives a wide base, so they can take sharp bends fast without overturning. Luggage piled on a bus's roof raises its CG, so a smaller tilt, on a bend or a sideways slope, is enough to overturn it.

4 k. $T = 2\pi\sqrt{(l/g)}$, so a smaller g gives a longer period, fewer swings in an hour, and a clock that runs slow.
In Kathmandu, g is very slightly smaller, because it is farther from the Earth's center, so the clock runs slightly slow (by about 18 seconds a day: g falls by about 0.04 %, and T grows by half that, about 0.02 %; Kathmandu being a little farther north, and the rocks under the valley, change it a little). On the Moon, g is about a sixth of the Earth's, so the period is $\sqrt{6} \approx 2.4$ times as long and the clock runs very slow, about 2.4 times too slow.

5 a.

$$T = 2\pi\sqrt{\frac{2.5}{9.8}} = 6.28 \times 0.505 = 3.17$$

$f = \dfrac{1}{3.17} = 0.32$ Hz. **T ≈ 3.2 s, f ≈ 0.32 Hz.**

5 b. $T = 100 \div 50 = 2.0$ s.

$$l = \frac{gT^2}{4\pi^2} = \frac{9.8 \times 2.0^2}{4 \times 3.14^2} = \frac{39.2}{39.4} = 0.99$$

About 0.99 m: it is a seconds pendulum.

5 c. $T = 31.2 \div 20 = 1.56$ s.

$$g = \frac{4\pi^2 l}{T^2} = \frac{4 \times 3.14^2 \times 0.60}{1.56^2} = \frac{23.66}{2.434} = 9.72\ ^2$$

g ≈ 9.7 m/s².

5 d.

$$T_2 = T_1\sqrt{\frac{g_1}{g_2}} = 2 \times \sqrt{\frac{9.8}{2.45}} = 2 \times \sqrt{4} = 4$$

4 s: a quarter of the gravity, twice the period.

5 e. f = 300 ÷ 60 = 5 Hz, so T = 1 ÷ 5 = 0.2 s.

$$v = \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 0.60}{0.2} = 18.8$$

T = 0.2 s; the tip moves at about 19 m/s (68 km/h).

5 f. $T = 1 \div 2 = 0.5$ s; $v = \dfrac{2\pi r}{T} = \dfrac{2 \times 3.14 \times 1.0}{0.5} = 12.6$ m/s.

$$F = \frac{mv^2}{r} = \frac{0.5 \times 12.56^2}{1.0} = 78.9$$

12.6 m/s; a tension of about 79 N.

5 g.

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{1800 \times 40}{200}} = \sqrt{360} = 19$$

About 19 m/s (68 km/h). Faster than that, the road can't pull hard enough and the motorbike skids outwards.

5 h.

i. r = 6,400 + 3,200 = 9,600 km: $E = 9.8 \times \left(\dfrac{6400}{9600}\right)^2 = 9.8 \times \dfrac{4}{9} =$ **4.36 N/kg**.

ii. r = 12,800 km = 2R: $E = 9.8 \times \dfrac{1}{4} =$ **2.45 N/kg**.

5 i. $GM = 6.67 \times 10^{-11} \times 6.4 \times 10^{23} = 4.27 \times 10^{13}$ N m²/kg; $R = 3.4 \times 10^6$ m.

i. $E = \dfrac{GM}{R^2} = \dfrac{4.27 \times 10^{13}}{(3.4 \times 10^6)^2} = \dfrac{4.27 \times 10^{13}}{1.156 \times 10^{13}} =$ **3.7 N/kg**.

ii. $V = -\dfrac{GM}{R} = -\dfrac{4.27 \times 10^{13}}{3.4 \times 10^6} =$ **-1.26×10^7 J/kg**.

iii. $v_e = \sqrt{-2V} = \sqrt{2 \times 1.26 \times 10^7} = \sqrt{2.51 \times 10^7} =$ **5.0×10^3 m/s**, about 5.0 km/s.

5 j.

$$v_e = \sqrt{-2V} = \sqrt{2 \times 8.0 \times 10^6} = \sqrt{1.6 \times 10^7} = 4.0 \times 10^3$$

Kinetic energy needed: $m \times (-V) = 500 \times 8.0 \times 10^6 = 4.0 \times 10^9$ J (the same as ½ × 500 × 4000²).

4.0 km/s; 4.0×10^9 J.

5 k.

$$x = \frac{120 \times 50 + 30 \times 90}{120 + 30} = \frac{6000 + 2700}{150} = \frac{8700}{150} = 58$$

At the 58 cm mark, moved 8 cm from the middle towards the load.

5 l.

i. h = 1.8 ÷ 2 = 0.90 m: $\tan\theta = \dfrac{0.25}{0.90} = 0.278$, θ = **15.5°**.

ii. h = 0.60 m: $\tan\theta = \dfrac{0.25}{0.60} = 0.417$, θ = **22.6°**.

Putting heavy things low makes a tall cupboard harder to tip over, which matters in an earthquake: tall cupboards should also be fixed to the wall.