Unit 11 · Force

STUDY NOTES

Key terms

Simple pendulum: small, heavy bob on a light, inextensible thread from a fixed point; swings freely in a vertical plane.

Effective length l: point of suspension to **center** of bob = thread + bob's radius.

Amplitude: greatest distance of bob from mean position (or angle θ with the vertical).

Oscillation: one complete to-and-fro: O → A → O → B → O.

Time period T: time for one oscillation (s).
Frequency f: oscillations per second (Hz).

Restoring force: always pulls a body back towards its mean position.

Periodic motion: repeats after equal intervals of time.

Oscillatory motion: periodic, to and fro about a mean position.

Uniform circular motion: along a circle at constant speed.

Centripetal force: acts on a body moving in a circle, towards the center; keeps it in the circle.

Gravitational field: space round a mass in which another mass feels a gravitational force.

Field intensity E: gravitational force on a unit mass at a point.

Gravitational potential V: work done in bringing a unit mass from infinity to a point.

Escape velocity: smallest speed from a planet's surface to escape its field and never return (no air).

Center of gravity (G): point through which the whole weight acts, in any position.

Formulas

$T = 2\pi\sqrt{\frac{l}{g}}, g = \frac{4\pi^2 l}{T^2}$	l effective length (m); g (m/s^2) **Use:** swings up to about 10°
$f = \frac{1}{T}; v = \frac{2\pi r}{T}$	f (Hz); r radius (m) **Use:** speed in a circle
$a = \frac{v^2}{r}; F = \frac{mv^2}{r} = \frac{4\pi^2 mr}{T^2}$	m (kg); towards the center **Use:** centripetal acceleration, force
$E = \frac{F}{m} = \frac{GM}{r^2}$	r from the **center** (N/kg) **Use:** field; = g there
$V = -\frac{GM}{r}$	J/kg; scalar **Use:** potential; surface: $V = -gR$
$W = m(V_2 - V_1)$	m moved from V_1 to V_2 **Use:** work to lift far up
$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} = \sqrt{-2V}$	R planet's radius **Use:** escape velocity; $\frac{1}{2}v_e^2 = -V$
$\tan\theta = \frac{b/2}{h}$	b base width; h height of G **Use:** tilt at which a box topples

- $G = 6.67 \times 10^{-11}$ N m^2/kg^2; Earth: $M = 5.97 \times 10^{24}$ kg, $R = 6{,}371$ km, $GM = 3.98 \times 10^{14}$ N m^2/kg

Simple pendulum

- mean position: tension = weight, at rest
- at θ: $mg\cos\theta$ along thread, balanced by tension T; $mg\sin\theta$ across it = restoring force
- at O: restoring force 0, speed greatest; inertia carries it on

- extremes: all PE; mean: all KE; a little lost each swing, so amplitude shrinks; a clock adds a small push

Laws

1. **Isochronism**: small amplitudes: T doesn't depend on amplitude
2. **Mass**: T doesn't depend on mass or material of bob
3. **Length**: $T \propto \sqrt{l}$
4. **Acceleration**: $T \propto 1/\sqrt{g}$

- mass cancels: bigger pull, but harder to speed up, same factor
- big swings a little longer: 0.05% at 5°, 0.4% at 15°, 1.7% at 30°
- **seconds pendulum**: $T = 2$ s, $l \approx 0.99$ m
- $l \times 4 \to T \times 2$; $l \div 4 \to T \div 2$
- clock in summer: rod expands, $l \uparrow$, $T \uparrow \to$ runs **slow**; on a mountain, $g \downarrow \to$ slow
- finding g: time 20 oscillations (error of the watch 20 × smaller in T); swing $\leq 10°$; start at mean position; T^2 against l: straight line through origin, slope $\approx$ 4.0 s^2/m, $g = 4\pi^2 \div$ slope

Periodic motion

- T:
  - second hand 60 s
  - hour hand 12 h
  - Earth's spin 1 day
  - round the Sun 365¼ days
  - new moon to new moon 29.5 days
  - heartbeat (72 a minute) 0.83 s
- **oscillatory**:
  - pendulum
  - ping
  - spring
  - guitar string
  - sewing machine needle
- **periodic only** (round and round):
  - clock hands
  - fan blades
  - Earth round the Sun
- every oscillatory motion is periodic; not the reverse
- **non-periodic**:
  - bouncing ball
  - bus in traffic
  - falling leaf

Circular motion and centripetal force

- speed constant, **velocity changes** (direction): always accelerating, towards the center
- velocity along the **tangent**; water off an umbrella, mud off a tyre, sparks off a grinder fly along it
- same T on a wheel: farther out $\to$ faster
- $F \propto m$; $F \propto v^2$ ($v \times 2 \to F \times 4$); $F \propto 1/r$
- not a new force: a job done by a real force
- stone on a string: tension
- bus on a bend: tyres' friction
- Moon, satellite: Earth's gravity
- planets: Sun's gravity
- electron (Bohr): electric attraction
- clothes in a drum: drum's wall
- **centrifugal**: pseudo force, size mv^2/r, away from center, felt only going round with the body; **not** the reaction (reaction to a string's pull acts on the hand)
- bus on a bend: body goes on straight (inertia); bus turns away; seat pushes it inwards
- **uses**:
  - banked roads (road's slanted push helps)
  - cyclist leans in
  - slow signs (half speed, quarter force)
  - spin dryers
  - cream separators, blood centrifuges
  - hammer throw
  - satellites

Gravitational field

- field lines point to the Earth's center; closer near the surface = stronger
- E is a **vector** towards the mass; = g at that point; 1 N/kg = 1 m/s^2
- $r \times 2 \to E \div 4$; $r \times 3 \to E \div 9$
- height R above the surface means $r = 2R$

Place	r (km)	E (N/kg)
sea level	6,371	9.81
top of Sagarmatha	6,380	9.78
Space Station	6,771	8.69
$2R$	12,742	2.45
geostationary	42,157	0.224
Moon's distance	384,400	0.0027

- Moon's surface: $E = 1.62$ N/kg, about ⅙ of the Earth's

- astronaut at the Station: pull ≈ 610 N; floats because she and the Station fall together
- fields of Earth and Moon cancel ≈ 346,000 km from the Earth

Potential and escape velocity

- zero at **infinity**; negative near the Earth: the field does the work as a mass comes in
- **scalar**, J/kg; rises towards 0 far away
- surface: $V = -6.25 \times 10^7$ J/kg $= -62.5$ MJ/kg; at 2 R: -31.3 MJ/kg
- PE of mass m: $U = mV = -GMm/r$
- mgh fails far up: g weakens (to $2R$: gives twice too much)

Relation with escape velocity

- needs a gain of GMm/R, from KE: $\frac{1}{2}mv_e^2 = GMm/R$
- m cancels: $v_e = \sqrt{2GM/R}$; since $V = -GM/R$: $v_e = \sqrt{-2V}$
- deeper potential → greater v_e
- independent of body's mass and of direction; only the planet's M, R
- v_e in km/s:
  - Moon 2.38
  - Earth 11.2
  - Sun 618
  - Mars 5.03
  - Jupiter 59.5
- orbit just above the surface: 7.9 km/s; $v_e = \sqrt{2} \times$ that
- Moon has no air: low v_e, fast molecules leak away

Center of gravity and stability

- uniform:
  - rod → midpoint
  - rectangle → diagonals cross
  - triangle → medians cross
  - disc, ring, sphere → center
  - cylinder → middle of axis
  - solid cone → on axis, ¼ of height above base
- may lie outside the body:
  - ring
  - bangle
  - boomerang
- uneven body: nearer the heavy part (hammer: near its head)
- **finding it**:
  - long thin → balance
  - sheet → hang from a hole with a plumb line, draw the line
  - second hole, second line
  - G where they cross
  - third hole checks
- hanging freely, G comes to rest below the support
- meter stick + load: $x =$ (sum of mass × position) ÷ total mass

Equilibrium	Tilt	Then	Example
stable	raises G	falls back	cone on base
unstable	lowers G	falls over	cone on tip
neutral	same height	stays	ball, cone on side

- **topples** when the vertical from G falls **outside the base**
- more stable: **low G, wide base** → must tilt farther
- racing cars low and wide
- heavy loads low, not on the roof
- lamps, cylinders have heavy, wide bases
- feet apart in a bus
- porter leans forward with a doko
- tumbler doll, ship's ballast

COMMON MISTAKES

✗ thread alone as l
✓ add the bob's radius; thread + **radius**.

✗ A to B as one oscillation
✓ that is half; A → B → A.

✗ $T \propto l$, so $T \times 2$ needs $l \times 2$
✓ $T \propto \sqrt{l}$: $l \times$ **4**.

✗ constant speed, so no acceleration
✓ direction changes: accelerating, to the center.

✗ centrifugal keeps it in the circle, or is the reaction
✓ **centripetal**; centrifugal is a pseudo force.

✗ height R as $r = R$
✓ $r =$ **2**R from the center.

✗ rockets leave the pad at 11.2 km/s
✓ speed of a body thrown with no more push.