Unit 12 · Continuity

WORKSHEET

Full marks 28

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 28

1 Key words [3]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

continuous **equal** **equal to zero** **indeterminate form** **integers** **left-hand limit** **open circle** **real numbers**

A curve that can be drawn without lifting the pen is a _____ curve. The value $f(x)$ heads for as x comes towards c from values less than c is the _____. When the left-hand and right-hand limits are _____, the limit exists. On a graph, a point missing from the curve is drawn as an _____. The form $\frac{0}{0}$ is called an _____: it is not a number. Unlike the natural numbers, the _____ leave no gaps on the number line.

2 Multiple choice [4]

Choose the correct option for each of the following questions.

i. What does $\lim_{x \to 3^+} f(x)$ mean? [1]

a. the value of $f(x)$ at $x = 3$

b. what $f(x)$ heads for as x comes towards 3 from values greater than 3

c. what $f(x)$ heads for as x comes towards 3 from values less than 3

d. the largest value of $f(x)$

ii. For a function, $\lim_{x \to 2^-} f(x) = 5$, $\lim_{x \to 2^+} f(x) = 5$, and $f(2)$ is not defined. Which is true? [1]

a. the limit doesn't exist

b. $f(x)$ is continuous at $x = 2$

c. the limit is 5, but $f(x)$ is discontinuous at $x = 2$

d. $f(2) = 0$

iii. $f(x) = 3x - 2$. What is $f(1.999)$? [1]

a. 3.997 **b.** 3.999

c. 4.003 **d.** 1

iv. Which of these graphs is continuous at $x = 1$? [1]

a. $y = \frac{x^2 - 1}{x - 1}$

b. $y = x^2 + 1$

c. $y = x$ for $x \leq 1$, $y = x + 1$ for $x > 1$

d. $y = 2x$ for $x \neq 1$, $y = 5$ for $x = 1$

3 Very short answers [3]

Write very short answers to the following questions.

a. Write, in symbols, the condition for $f(x)$ to be continuous at $x = a$. [1]

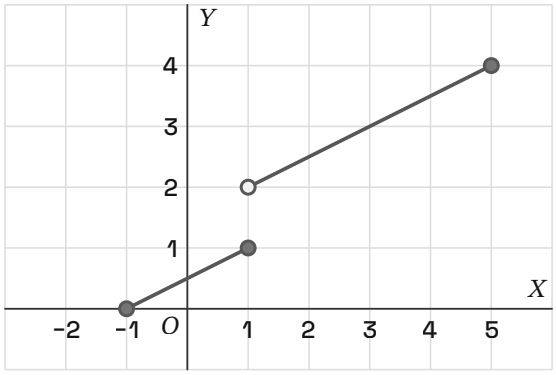
b. Is a wire bent sharply at a corner, but not broken, continuous at the corner? [1]

c. Find $f(-3)$ for $f(x) = \dfrac{x^2 - 9}{x + 3}$, and say whether it is defined. [1]

4 Read the graph [2]

Look at the graph and answer.

Write the intervals where this graph is continuous and the point where it is discontinuous.



5 Short answers [6]

Write short answers to the following questions.

a. Find the mistake. For $f(x) = 2x + 1$ for $x \leq 3$ and $4x - 5$ for $x > 3$, Bina writes: "$\lim_{x \to 3^-} f(x) = 7$ and $\lim_{x \to 3^+} f(x) = 7$, so $f(x)$ is continuous at $x = 3$." What has she left out? Is her conclusion right? [2]

b. $g(x) = 5 - 2x$. Find $g(0.99)$, $g(0.999)$, $g(1.01)$ and $g(1.001)$, and write $\lim_{x \to 1} g(x)$. **[2]**

c. Test the continuity of $f(x) = \dfrac{x^2 - 4}{x - 2}$ at $x = 2$ by putting in values of x. **[2]**

6 Long answers **[8]**

Solve the following problems. Show your working.

a. $p(x) = 3x - 1$ for $1 \leq x \leq 2$ and $p(x) = x + 3$ for $x > 2$. **[4]**

a. Find $p(1.99)$.
b. Find $p(2.01)$.
c. Find $\lim_{x \to 2^-} p(x)$ and $\lim_{x \to 2^+} p(x)$.
d. Is $p(x)$ continuous at $x = 2$? Give a reason.

b. $f(x) = 2x + k$ for $x < 3$, $f(x) = 9$ for $x = 3$ and $f(x) = 4x - 3$ for $x > 3$. **[4]**

a. Find $\lim_{x \to 3^+} f(x)$.
b. Find $\lim_{x \to 3^-} f(x)$ in terms of k.
c. Find k so that $f(x)$ is continuous at $x = 3$.
d. With this k, is there any other x where $f(x)$ is discontinuous? Give a reason.

7 Think **[2]**

Think, then answer.

Ram says: "If I put $x = c$ into a function and get a number, the function must be continuous at c."
Is he right? Give an example.

8 Activity: Closer and closer (not marked)

Use a calculator. For each function, fill in the values just below and just above the point, then write what the values head for and whether the function is continuous there.

Function, point	$x = c$ − 0.01	$x = c$ − 0.001	$x = c$ + 0.001	$x = c$ + 0.01	Limit	$f(c)$	Continuous?
x^2, $c = 3$							
$\frac{x^2 - 1}{x - 1}$, $c = 1$							
$\frac{1}{x}$, $c = 2$							