Unit 12 · Continuity

HARDER WORKSHEET

Full marks 24

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 24

1 Multiple choice [3]

Choose the correct option for each of the following questions.

i. $f(x) = \dfrac{x^2 - 5x + 6}{x - 3}$. What is $\lim_{x \to 3} f(x)$? [1]

a. 0 **b.** 1

c. 3 **d.** it doesn't exist, since $f(3) = \dfrac{0}{0}$

ii. Where is $f(x) = \dfrac{x + 1}{(x - 2)(x + 4)}$ discontinuous? [1]

a. only at $x = -1$ **b.** only at $x = 2$

c. at $x = 2$ and $x = -4$ **d.** nowhere

iii. $h(x) = 2x + a$ for $x \leq 1$ and $ax + 5$ for $x > 1$. For which a is $h(x)$ continuous at $x = 1$? [1]

a. $a = 1$ **b.** $a = 3$

c. every value of a **d.** no value of a

2 Short answers [11]

Write short answers to the following questions.

a. $f(x) = ax + 1$ for $x < 2$, $f(x) = 5$ for $x = 2$ and $f(x) = x^2 + b$ for $x > 2$. [3]
Find a and b so that $f(x)$ is continuous at $x = 2$.

b. Show that $f(x) = \dfrac{x^3 - 8}{x - 2}$ is discontinuous at $x = 2$, and find the value $f(2)$ should be given to make it continuous there. [3]

c. $g(x) = \dfrac{|x-2|}{x-2}$. Find $g(1.9)$, $g(1.99)$, $g(2.1)$ and $g(2.01)$. Does $\lim_{x \to 2} g(x)$ exist? **[2]**

d. Work backwards. $f(x) = kx^2$ for $x \leq 2$ and $3x + k$ for $x > 2$, and $f(x)$ is continuous at $x = 2$. Find k, then find $f(2)$ and $f(2.1)$. **[3]**

3 Long answers **[8]**

Solve the following problems. Show your working.

a. A tank is filled from two taps. Its water level, in cm, t minutes after the start, is $h(t) = 2t$ for $0 \leq t \leq 5$ and $h(t) = t + 5$ for $5 < t \leq 10$. (the second tap is closed at $t = 5$). The numbers are made up. **[4]**

a. Find $h(4.9)$, $h(5)$ and $h(5.1)$.
b. Find $\lim_{t \to 5^-} h(t)$ and $\lim_{t \to 5^+} h(t)$.
c. Is $h(t)$ continuous at $t = 5$? Give a reason.
d. The graph has a corner at $t = 5$. What does the corner mean for the water, and why could the level never have a jump?

b. Make your own. **[4]**

a. Write a function in two pieces, each a straight line, with different slopes, that is continuous at $x = -1$. Show that it is.

b. Change one value of your function so that it is discontinuous at $x = -1$, though its limit there still exists.

4 Think **[2]**

Think, then answer.

A car park charges Rs 50 for every hour or part of an hour. Sketch the charge against the time parked, from 0 to 4 hours. Where is it discontinuous? Is the time parked continuous?