Unit 12 · Continuity

HARDER WORKSHEET · ANSWER KEY

Full marks 24

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Multiple choice

i. **b.** 1 — A · 1

$x^2 - 5x + 6 = (x - 2)(x - 3)$, so for $x \neq 3$, $f(x) = x - 2$, which heads for 1. The limit exists even though $f(3)$ doesn't.

ii. **c.** at $x = 2$ and $x = -4$ — U · 1

It is not defined where the bottom is 0: $x = 2$ and $x = -4$. At $x = -1$ its value is 0, which is a perfectly good value.

iii. **d.** no value of a — HA · 1

LHL $= 2 + a$ and RHL $= a + 5$. They would be equal only if $2 = 5$, so no a joins the pieces: the jump is always 3.

2 Short answers

a. MARKING SCHEME — A · 3

- ✓ LHL $= 2a + 1$ and RHL $= 4 + b$.
- ✓ $a = 2$.
- ✓ $b = 1$.

b. ANSWER — HA · 3

$f(2) = \dfrac{8 - 8}{0} = \dfrac{0}{0}$, not defined, so f is discontinuous at 2.

For $x \neq 2$, $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$, so $f(x) = x^2 + 2x + 4$, which heads for $4 + 4 + 4 = 12$ from both sides ($f(1.999) \approx 11.994$, $f(2.001) \approx 12.006$).

Defining $f(2) = 12$ fills the hole: then LHL = RHL = $f(2) = 12$, continuous.

MARKING SCHEME

- ✓ $f(2) = \dfrac{0}{0}$, not defined: discontinuous.
- ✓ $\lim_{x \to 2} f(x) = 12$ (by factorizing or by values).
- ✓ $f(2) = 12$ makes it continuous.

c. ANSWER — HA · 2

Below 2, $|x - 2| = -(x - 2)$, so $g(1.9) = g(1.99) = -1$. Above 2, $g(2.1) = g(2.01) = 1$. LHL $= -1$ and RHL $= 1$ are different, so **the limit does not exist**: the graph jumps from -1 to 1 at $x = 2$.

MARKING SCHEME

- ✓ The values $-1, -1, 1, 1$.
- ✓ LHL $\neq$ RHL, so no limit.

d. MARKING SCHEME — A · 3

- ✓ $4k = 6 + k$.
- ✓ $k = 2$.
- ✓ $f(2) = 8$ and $f(2.1) = 8.3$.

3 Long answers

a. ANSWER

a. $h(4.9) = 9.8$, $h(5) = 10$, $h(5.1) = 10.1$.

b. LHL $= 2 \times 5 = 10$; RHL $= 5 + 5 = 10$.

c. Yes: LHL = RHL = $h(5) = 10$.

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking

d. The corner is where the level starts rising more slowly (1 cm a minute instead of 2), because one tap was closed. A jump would mean the water was suddenly 10 cm deep and then 14 cm deep with no time between: water can't appear at once, so the level must pass through every height in between. **A · 4**

MARKING SCHEME

- ✓ The three values 9.8, 10, 10.1.
- ✓ LHL = RHL = 10.
- ✓ $h(5) = 10$, so continuous.
- ✓ The corner: slower rising; no jump because the level passes every height.

b. **ANSWER** **HA · 4**

a. One answer: $f(x) = 2x + 5$ for $x \leq -1$ and $-x + 2$ for $x > -1$. LHL $= -2 + 5 = 3$, RHL $= 1 + 2 = 3$, $f(-1) = 3$: continuous. (Any two lines through the same point at $x = -1$ will do.)

b. Keep both pieces but define $f(-1) = 0$ separately: the limit is still 3, but $f(-1) = 0 \neq 3$, so it is discontinuous (a moved point).

MARKING SCHEME

- ✓ Two lines with different slopes that meet at $x = -1$.
- ✓ LHL = RHL = $f(-1)$ shown.
- ✓ A changed value at $x = -1$ only.
- ✓ Limit unchanged, value different, so discontinuous.

4 Think

ANSWER **HA · 2**

The charge is Rs 50 from just after 0 up to 1 hour, Rs 100 after 1 hour up to 2, Rs 150 up to 3, Rs 200 up to 4: flat steps, each with a filled dot at its right end and an open circle at the left. It is discontinuous at 1, 2 and 3 hours, where it jumps by Rs 50 (and at 0, where it jumps from nothing to Rs 50 as soon as the car is parked). The time parked itself is continuous: it passes through every value; only the charge counts in whole hours.

MARKING SCHEME

- ✓ Steps of Rs 50, jumping at whole hours.
- ✓ Discontinuous at 1, 2 and 3 hours; the time itself is continuous.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	1	11	12	**24**

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking