Unit 12 · Continuity

ANSWER KEY · FOR TEACHERS

Full marks 28

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. continuous
2. left-hand limit
3. equal
4. open circle
5. indeterminate form
6. real numbers

Each carries 0.5 mark. K · 3

2 Multiple choice

i. **b.** what $f(x)$ heads for as x comes towards 3 from values greater than 3 K · 1

The plus sign means from above, the right-hand side. The limit never uses $f(3)$ itself.

ii. **c.** the limit is 5, but $f(x)$ is discontinuous at $x = 2$ U · 1

The two sides agree, so the limit is 5. But $f(2)$ must be defined and equal 5 for continuity: here the graph has a hole.

iii. **a.** 3.997 A · 1

$3 \times 1.999 = 5.997$, and $5.997 - 2 = 3.997$.

iv. **b.** $y = x^2 + 1$ U · 1

$x^2 + 1$ is a polynomial: continuous everywhere. The first has a hole at 1, the third a jump, the fourth a point moved off the line.

3 Very short answers

a. MARKING SCHEME K · 1

✓ $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.

b. MARKING SCHEME U · 1

✓ Yes, continuous (no break at the bend).

c. MARKING SCHEME A · 1

✓ $\frac{0}{0}$, not defined.

4 Read the graph

MARKING SCHEME U · 2

✓ Discontinuous at $x = 1$.

✓ Continuous on $-1 \leq x \leq 1$ and $1 < x \leq 5$.

5 Short answers

a. MARKING SCHEME U · 2

✓ $f(3)$ must be found and compared with the limits.

✓ $f(3) = 7$, so continuous.

b. MARKING SCHEME A · 2

✓ The four values 3.02, 3.002, 2.98, 2.998.

✓ $\lim_{x \to 1} g(x) = 3$.

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c. MARKING SCHEME A · 2

- ✓ $\lim_{x \to 2} f(x) = 4$ from values on both sides.
- ✓ $f(2)$ not defined, so discontinuous.

6 Long answers

a. ANSWER A · 4

a. $p(1.99) = 3 \times 1.99 - 1 = 4.97$.
b. $p(2.01) = 2.01 + 3 = 5.01$.
c. $\lim_{x \to 2^-} p(x) = 6 - 1 = 5$; $\lim_{x \to 2^+} p(x) = 2 + 3 = 5$.
d. $p(2) = 3 \times 2 - 1 = 5$. LHL = RHL = $p(2) = 5$, so $p(x)$ is continuous at $x = 2$.

MARKING SCHEME

- ✓ $p(1.99) = 4.97$.
- ✓ $p(2.01) = 5.01$.
- ✓ LHL = 5 and RHL = 5.
- ✓ $p(2) = 5$, all equal, so continuous.

b. ANSWER HA · 4

a. $4 \times 3 - 3 = 9$.
b. $2 \times 3 + k = 6 + k$.
c. LHL = RHL = $f(3)$: $6 + k = 9$, so $k = 3$; and $f(3) = 9$. ✓
d. No. Each piece is a straight line, continuous wherever it applies, so the only place a break could be is the joint at $x = 3$, which is now joined.

MARKING SCHEME

- ✓ RHL = 9.
- ✓ LHL = $6 + k$.
- ✓ $k = 3$.
- ✓ No other break: each piece is a line; the joint is joined.

7 Think

ANSWER HA · 2

No. A value at c is only one of the three conditions. Example: $f(x) = x + 1$ for $x \leq 1$ and $x + 3$ for $x > 1$: $f(1) = 2$, but from the right $f(x)$ heads for 4, so the graph jumps and f is discontinuous at 1.

MARKING SCHEME

- ✓ No: the limits must exist and equal $f(c)$ too.
- ✓ A correct example with $f(c)$ defined and a break at c.

8 Activity: Closer and closer

(not marked)

WHAT STUDENTS SHOULD FIND

Function, point	Values	Limit	$f(c)$	Continuous?
x^2, $c = 3$	8.9401, 8.994001, 9.006001, 9.0601	9	9	yes
$\frac{x^2 - 1}{x - 1}$, $c = 1$	1.99, 1.999, 2.001, 2.01	2	0/0, not defined	no
$\frac{1}{x}$, $c = 2$	0.50251..., 0.50025..., 0.49975..., 0.49751...	0.5	0.5	yes

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	7	10	6	**28**

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