Unit 11 · Statistics

HARDER WORKSHEET · ANSWER KEY

Full marks 26

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Multiple choice

i. **c.** it does not change — U · 1

The mean and the standard deviation are both multiplied by 3, so $\frac{\sigma}{\bar{X}}$ stays the same. The C.V. doesn't depend on the unit: marks out of 50 or the same marks out of 150.

ii. **c.** 8.33% — A · 1

$\sigma = 10\%$ of $50 = 5$, and adding 10 to every value leaves it at 5. The new mean is 60, so the C.V. is $\frac{5}{60} \times 100\% \approx 8.33\%$.

iii. **b.** the values in the third quarter are more spread out than those in the second — HA · 1

A quarter of the values lies between Q_1 and the median, and another quarter between the median and Q_3: the same number of values. If the second stretch is longer, those values are more spread out.

2 Short answers

a. **MARKING SCHEME** — A · 3

- ✓ $x = 8$.
- ✓ $\sigma = \sqrt{5} \approx 2.24$.
- ✓ C.V. $\approx 44.7\%$.

b. **ANSWER** — HA · 3

a. $\Sigma fm = 20 + 15k + 150 + 140 = 310 + 15k$ and $N = 14 + k$. $310 + 15k = 19(14 + k) = 266 + 19k$, so $4k = 44$ and $k = \mathbf{11}$.

b. $N = 25$. $\Sigma fm^2 = 4 \times 25 + 11 \times 225 + 6 \times 625 + 4 \times 1225 = 11225$.

$\sigma = \sqrt{\frac{11225}{25} - 19^2} = \sqrt{449 - 361} = \sqrt{88} \approx \mathbf{9.38}$.

MARKING SCHEME

- ✓ $\Sigma fm = 310 + 15k$ and $N = 14 + k$ set equal to the mean.
- ✓ $k = 11$.
- ✓ $\sigma = \sqrt{88} \approx 9.38$.

c. **ANSWER** — HA · 2

No. The standard deviation is never smaller than the mean deviation from the mean: squaring makes the larger distances count for more, so the root of the mean of the squares is at least the mean of the distances. (They are equal only when every value is the same distance from the mean.)

MARKING SCHEME

- ✓ Not possible.
- ✓ $\sigma \geq$ MD, with a reason.

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d. **ANSWER** HA · 3

The mean is $\frac{a+b}{2}$. Each number is $\frac{|a-b|}{2}$ from it: $a - \frac{a+b}{2} = \frac{a-b}{2}$ and $b - \frac{a+b}{2} = -\frac{a-b}{2}$.

So both squared distances are $\frac{(a-b)^2}{4}$, and their mean is $\frac{(a-b)^2}{4}$.

$\sigma = \sqrt{\frac{(a-b)^2}{4}} = \frac{|a-b|}{2}$. Hence shown. (For 30 and 70, $\sigma = 20$.)

MARKING SCHEME

- ✓ The mean $\frac{a+b}{2}$ and the two distances $\pm\frac{a-b}{2}$.
- ✓ The mean of the squares $\frac{(a-b)^2}{4}$.
- ✓ Its square root $\frac{|a-b|}{2}$.

e. **MARKING SCHEME** A · 2

- ✓ Both C.V.s: 20% and 18.2%.
- ✓ Gita, as her C.V. is smaller.

3 Long answers

a. **ANSWER** A · 4

a. 4,000–6,000: 6; 6,000–8,000: 12; 8,000–10,000: 12; 10,000–12,000: 7; 12,000–14,000: 3.
In thousands, with $A = 9$ and $h = 2$: $d' = -2, -1, 0, 1, 2$; $fd' = -12, -12, 0, 7, 6$, $\Sigma fd' = -11$; $fd'^2 = 24$, 12, 0, 7, 12, $\Sigma fd'^2 = 55$.

b. $\overline{X} = 9 + \frac{-11}{40} \times 2 = 9 - 0.55 = 8.45$ thousand, **Rs 8,450**.

c. $\sigma = \sqrt{\frac{55}{40} - \left(\frac{-11}{40}\right)^2} \times 2 = \sqrt{1.375 - 0.0756} \times 2 = \sqrt{1.2994} \times 2 \approx 2.28$ thousand, **about Rs 2,280**.

d. C.V. $= \frac{2.28}{8.45} \times 100\% \approx$ **26.98%**.

MARKING SCHEME

- ✓ The class frequencies 6, 12, 12, 7, 3.
- ✓ $\overline{X}$ = Rs 8,450.
- ✓ $\sigma \approx$ Rs 2,280.
- ✓ C.V. $\approx$ 27%.

b. **ANSWER** HA · 4

a. One answer: 35, 45, 50, 55, 65. Mean $\frac{250}{5} = 50$; distances $-15, -5, 0, 5, 15$; squares 225, 25, 0, 25, 225, total 500; $\sigma = \sqrt{\frac{500}{5}} = \sqrt{100} =$ **10**. (Any five marks with distances adding to 0 and squares adding to 500 will do.)

b. For 35, 45, 50, 55, 65: MD $= \frac{15+5+0+5+15}{5} =$ **8**, less than 10.

c. The mean stays 50, the squares still add up to 500, but now over 6 marks: $\sigma = \sqrt{\frac{500}{6}} = \sqrt{83.33} \approx$ **9.13**.

d. The new mark is exactly at the mean, so it adds nothing to the squared distances but one more to N: the average squared distance gets smaller, so the spread per student is less.

MARKING SCHEME

- ✓ Five marks with mean 50 and $\sigma = 10$, checked.
- ✓ Their mean deviation found and less than 10.
- ✓ New $\sigma = \sqrt{500/6} \approx 9.13$ (or the right value for the student's own marks).
- ✓ The reason: a value at the mean adds to N but not to the squares.

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4 Think

ANSWER HA · 2

The **quartile deviation**: the open class has no mid-value, so the mean and the standard deviation can't be found, but the quartiles can. Even with the class closed, a few very rich households would raise the standard deviation a lot (it squares the large distances), while the quartile deviation, which ignores the top quarter, would describe the typical households better.

MARKING SCHEME

- ✓ The quartile deviation, as the open class has no mid-value.
- ✓ Extreme incomes raise σ a lot; the Q.D. ignores them.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	1	10	15	**26**

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