Unit 10 · Vector

HARDER WORKSHEET · ANSWER KEY

Full marks 26

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Multiple choice

i. **a.** 0 **A · 1**

$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2$: $25 = 9 + 2\vec{a}\cdot\vec{b} + 16$, so $\vec{a}\cdot\vec{b} = 0$. The sides 3, 4, 5 make a right angle.

ii. **a.** They point in the same direction. **U · 1**

$\cos\theta = 1$ only at $\theta = 0°$: the same direction, whatever their lengths.

iii. **a.** (7, 9) **A · 1**

$P = B + \overrightarrow{AB} = (4, 5) + (3, 4) = (7, 9)$: it divides AB externally in the ratio 2 : 1, and $\frac{2(4, 5) - (1, 1)}{1} = (7, 9)$.

$(-2, -3)$ is the same distance beyond A.

2 Short answers

a. **MARKING SCHEME** **A · 3**

- ✓ The point written with the ratio k:1.
- ✓ $k = 3$, so the ratio 3 : 1.
- ✓ The point (4, 0).

b. **ANSWER** **HA · 3**

$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} = |\vec{a}|^2 - |\vec{b}|^2$. This is 0, so $|\vec{a}|^2 = |\vec{b}|^2$ and $|\vec{a}| = |\vec{b}|$, lengths being positive.

In a parallelogram on $\vec{a}$ and $\vec{b}$, the diagonals are $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$. If they are perpendicular, the two sides are equal: **the parallelogram is a rhombus**.

MARKING SCHEME

- ✓ $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$.
- ✓ So $|\vec{a}| = |\vec{b}|$.
- ✓ The diagonals are $\vec{a} \pm \vec{b}$, so the parallelogram is a rhombus.

c. **MARKING SCHEME** **A · 2**

- ✓ $W = 60$ J.
- ✓ The 8 N across the motion does no work.

d. **ANSWER** **HA · 3**

$\vec{a} + k\vec{b} = (3 + k)\vec{i} + 4\vec{j}$, so $|\vec{a} + k\vec{b}|^2 = (3 + k)^2 + 16$, least when $3 + k = 0$: $k = \mathbf{-3}$, length 4.

Then $\vec{a} + k\vec{b} = 4\vec{j}$, and $4\vec{j}\cdot\vec{i} = 0$: perpendicular to $\vec{b}$. The shortest way from the tip of $\vec{a}$ to the line of $\vec{b}$ meets it at a right angle.

MARKING SCHEME

- ✓ $|\vec{a} + k\vec{b}|^2 = (3 + k)^2 + 16$.
- ✓ $k = -3$.
- ✓ $4\vec{j}\cdot\vec{i} = 0$, so perpendicular.

e. **MARKING SCHEME** **A · 2**

- ✓ A vector 5 long, not along an axis (e.g. $3\vec{i} + 4\vec{j}$), with a second one perpendicular to it.
- ✓ The scalar product checked as 0.

3 Long answers

a. **ANSWER**

a. In a parallelogram $\overrightarrow{AD} = \overrightarrow{BC} = (-2, 4)$, so $D = (1 - 2, 1 + 4) = \mathbf{(-1, 5)}$.

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b. $\overrightarrow{AB} = (4, 2), \overrightarrow{BC} = (-2, 4)$: $\overrightarrow{AB} \cdot \overrightarrow{BC} = -8 + 8 = 0$, a right angle at B. A parallelogram with a right angle is a rectangle. **A · 4**

c. **Yes**: $|\overrightarrow{AB}| = |\overrightarrow{BC}| = \sqrt{20}$, so two neighboring sides are equal: a rectangle with equal sides is a square.

d. $\overrightarrow{AC} = (2, 6), \overrightarrow{BD} = (-6, 2)$: $-12 + 12 = 0$, perpendicular.

MARKING SCHEME

- ✓ $D(-1, 5)$.
- ✓ $\overrightarrow{AB} \cdot \overrightarrow{BC} = 0$, so a rectangle.
- ✓ $AB = BC = \sqrt{20}$, so a square.
- ✓ $\overrightarrow{AC} \cdot \overrightarrow{BD} = 0$.

b. **ANSWER** **HA · 4**

Let $ABCD$ be a rhombus, with $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AD} = \vec{b}$; all its sides are equal, so $|\vec{a}| = |\vec{b}|$.
$\overrightarrow{BC} = \overrightarrow{AD} = \vec{b}$, so by the triangle law $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \vec{a} + \vec{b}$, and $\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD} = \vec{b} - \vec{a}$.
$\overrightarrow{AC} \cdot \overrightarrow{BD} = (\vec{b} + \vec{a}) \cdot (\vec{b} - \vec{a}) = |\vec{b}|^2 - |\vec{a}|^2 = 0$.
So $AC \perp BD$. Hence proved.

MARKING SCHEME

- ✓ $\overrightarrow{AB} = \vec{a}, \overrightarrow{AD} = \vec{b}$ with $|\vec{a}| = |\vec{b}|$.
- ✓ $\overrightarrow{AC} = \vec{a} + \vec{b}$ and $\overrightarrow{BD} = \vec{b} - \vec{a}$.
- ✓ $\overrightarrow{AC} \cdot \overrightarrow{BD} = |\vec{b}|^2 - |\vec{a}|^2$.
- ✓ $= 0$, so the diagonals are perpendicular.

4 Think

ANSWER **HA · 2**

No. $\vec{a} \cdot \vec{b} = 4 \times 4 \times \cos\theta = 16\cos\theta$, and $\cos\theta$ is at most 1, so the product is at most 16 (forces in the same direction). 20 is impossible.

MARKING SCHEME

- ✓ No.
- ✓ The product is at most $|\vec{a}|\,|\vec{b}| = 16$.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	1	13	12	**26**

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