Unit 10 · Vector

ANSWER KEY · FOR TEACHERS

Full marks 30

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. scalar product
2. scalar
3. perpendicular
4. position vector
5. triangle law
6. midpoint

K · 3

Each carries 0.5 mark.

2 Read the figure

1. F sin θ
2. F cos θ

U · 1

Each carries 0.5 mark.

3 Multiple choice

i. **b.** 5 — K · 1

$\vec{a} \cdot \vec{b} = 2 \times 5 \times \cos 60^\circ = 10 \times \frac{1}{2} = 5$. 10 forgets the cosine; $5\sqrt{3}$ uses the sine.

ii. **a.** $2\vec{i} + 3\vec{j}$ and $3\vec{i} - 2\vec{j}$ — U · 1

$2 \times 3 + 3 \times (-2) = 0$. The second pair is parallel (one is twice the other), the last pair opposite.

iii. **a.** $\vec{i} + 2\vec{j}$ — A · 1

$\frac{1}{2}(4 - 2, -2 + 6) = \frac{1}{2}(2, 4) = (1, 2)$. The second forgets to halve; the last two subtract.

iv. **a.** $\vec{0}$ — U · 1

From A to B, to C and back to A: you end where you started, so the sum is the zero vector.

v. **a.** (2, 3) — A · 1

$\frac{1 \times (6, 9) + 2 \times (0, 0)}{3} = (2, 3)$, one third of the way from A. (4, 6) is the ratio 2 : 1, (3, 4.5) the midpoint.

4 Very short answers

a. K · 1

MARKING SCHEME

✓ $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$.

b. U · 1

MARKING SCHEME

✓ Obtuse.

c. A · 1

MARKING SCHEME

✓ $\vec{a} \cdot \vec{b} = 1$.

5 Short answers

a. U · 2

MARKING SCHEME

✓ $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ (end minus start).

✓ $\overrightarrow{AB} = (3, 4)$.

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b. MARKING SCHEME A · 2

- ✓ $6 = 4|\vec{b}|\cos 60°$ written.
- ✓ $|\vec{b}| = 3$.

c. MARKING SCHEME A · 2

- ✓ $\overrightarrow{OM} = \frac{12}{5}\vec{i} - \vec{j}$.
- ✓ $\overrightarrow{ON} = \frac{7}{2}\vec{i} + \frac{9}{2}\vec{j}$.

d. MARKING SCHEME U · 2

- ✓ No such point.
- ✓ A reason: $m - n = 0$, or a point outside is always nearer one end.

6 Long answers

a. ANSWER A · 4

a. $\overrightarrow{BA} = \begin{pmatrix} 1-5 \\ 2-4 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} 3-5 \\ 8-4 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

b. $(-4)(-2) + (-2)(4) = 8 - 8 = 0$, so $\angle ABC = 90°$: the triangle is right-angled at B.

c. $\overrightarrow{OD} = \frac{1}{2}(\vec{i} + 2\vec{j} + 3\vec{i} + 8\vec{j}) = 2\vec{i} + 5\vec{j}$.

d. $\overrightarrow{BD} = (2-5, 5-4) = (-3, 1)$, so $BD = \sqrt{10}$; $\overrightarrow{AC} = (2, 6)$, so $AC = \sqrt{40} = 2\sqrt{10}$. $BD = \frac{1}{2}AC$: the midpoint of the hypotenuse is as far from B as from A and C.

MARKING SCHEME

- ✓ $\overrightarrow{BA} = (-4, -2)$ and $\overrightarrow{BC} = (-2, 4)$.
- ✓ Scalar product 0, so $\angle ABC = 90°$.
- ✓ $\overrightarrow{OD} = 2\vec{i} + 5\vec{j}$.
- ✓ $BD = \sqrt{10}$ and $AC = 2\sqrt{10}$, so $BD = \frac{1}{2}AC$.

b. ANSWER A · 4

a. $\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$ for the segment from A to B.

b. $\vec{r} = \frac{3(\vec{i} - 3\vec{j}) + 2(2\vec{i} + \vec{j})}{5} = \frac{7}{5}\vec{i} - \frac{7}{5}\vec{j}$.

c. $\vec{p} \cdot \vec{r} = \frac{14}{5} - \frac{7}{5} = \frac{7}{5}$; $|\vec{p}| = \sqrt{5}$, $|\vec{r}| = \frac{7}{5}\sqrt{2}$; $\cos \angle POR = \frac{1}{\sqrt{10}}$, so $\angle POR \approx 71.57°$.

d. $\vec{q} \cdot \vec{r} = \frac{28}{5}$, $|\vec{q}| = \sqrt{10}$; $\cos \angle QOR = \frac{2}{\sqrt{5}}$, so $\angle QOR \approx 26.57°$. $\angle POR$ is larger: R is nearer Q. Together they make $\angle POQ \approx 98.13°$.

MARKING SCHEME

- ✓ The formula $\frac{m\vec{b} + n\vec{a}}{m+n}$.
- ✓ $\vec{r} = \frac{7}{5}\vec{i} - \frac{7}{5}\vec{j}$.
- ✓ $\angle POR \approx 71.57°$.
- ✓ $\angle QOR \approx 26.57°$, so $\angle POR$ is larger.

7 Think

ANSWER HA · 2

No. $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ means $\vec{a} \cdot (\vec{b} - \vec{c}) = 0$: $\vec{b} - \vec{c}$ need only be perpendicular to $\vec{a}$. For example $\vec{a} = \vec{i}$, $\vec{b} = 2\vec{i} + 5\vec{j}$, $\vec{c} = 2\vec{i} - 3\vec{j}$: both products are 2, but $\vec{b} \neq \vec{c}$. A scalar product can't be "cancelled".

MARKING SCHEME

- ✓ No: $\vec{b} - \vec{c}$ need only be perpendicular to $\vec{a}$.
- ✓ A correct example with $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{b} \neq \vec{c}$.

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8 Activity: Measure an angle, then compute it

(not marked)

WHAT STUDENTS SHOULD FIND

$\vec{a}$	$\vec{b}$	$\vec{a} \cdot \vec{b}$	$\cos\theta$	**Angle**
(3, 1)	(1, 3)	6	0.6	53.13°
(4, 0)	(2, 2)	8	0.7071	45°
(2, 3)	(−3, 2)	0	0	90°
(3, 1)	(−2, 2)	−4	−0.4472	116.57°

The measured angles should be within a degree or two of these. The product is positive for the acute angles, 0 for the right angle and negative for the obtuse one.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	5	8	15	2	**30**

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