Unit 9 · Transformation

HARDER WORKSHEET

Full marks 32

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 32

1 Multiple choice [4]

Choose the correct option for each of the following questions.

i. A figure is reflected in the line $y = x$ and then in the line $y = -x$. Which single transformation does both? [1]

a. a rotation of $+90^\circ$ about O
b. a rotation of 180° about O
c. a translation
d. the reflection in the X-axis

ii. E is the enlargement $E[(0, 0), 2]$ and R the reflection in the X-axis. Which points stay where they are under $R \circ E$? [1]

a. every point on the X-axis
b. only the origin
c. every point on the Y-axis
d. no point at all

iii. A 2×2 matrix maps the unit square to a parallelogram of area 6 that is a mirror image of the square (it has been flipped over). What is the determinant of the matrix? [1]

a. 6
b. -6
c. $\frac{1}{6}$
d. 36

iv. $E[(2, 2), 3]$ is followed by $E[(2, 2), k]$. Together they are a rotation of 180° about $(2, 2)$. What is k? [1]

a. -3
b. $-\frac{1}{3}$
c. $\frac{1}{3}$
d. -1

2 Short answers [15]

Write short answers to the following questions. Show your working.

a. Work backwards. R is the reflection in the line $y = -x$ and T the translation by $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$. Under $T \circ R$ the image of a point P is $P''(5, 3)$. Find P. [2]

b. Show that a translation by $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$ followed by the reflection in the line $x = 1$ is a single reflection, and find its mirror line. Then check your answer with the point $(-2, 5)$. **[3]**

c. R is the reflection in the line $y = x$ and T the translation by $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$. Find every point that stays where it is under $R \circ T$, or show that none does. What does your answer tell you about whether $R \circ T$ could be a single reflection or rotation? **[3]**

d. The matrix $M = \begin{pmatrix} a & 2 \\ 3 & b \end{pmatrix}$ maps $P(1, 2)$ to $P'(5, 9)$. **[3]**

i. Find a and b.

ii. A triangle of area 4 square units is transformed by M. Find the area of its image, and say whether the image is flipped over.

e. The matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ maps a point P to $P'(7, 4)$. Find P by using the inverse of M (Unit 6). **[2]**

f. Find the mistake. A is the reflection in the X-axis, $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, and B the rotation of $+90^\circ$ about O, $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Hari wants the single matrix for "first A, then B". He works out $AB = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ and says it is the reflection in $y = -x$. What is wrong, and what is the right answer? **[2]**

3 Long answers **[11]**

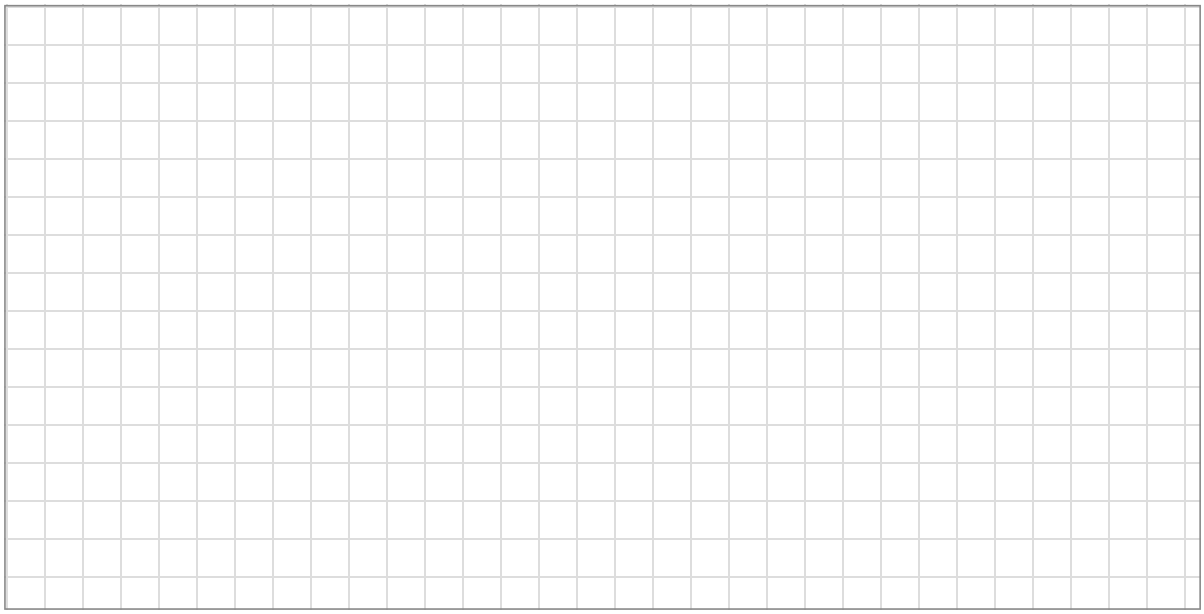
Solve the following problems. Show your working.

a. $\triangle KLM$ has vertices $K(2, 1)$, $L(4, 1)$ and $M(3, 3)$. E is the enlargement $E[(1, 0), -2]$ and R the reflection in the line $y = x$. **[4]**

i. Find the images of K, L and M under $R \circ E$.

ii. Draw $\triangle KLM$ and its final image on one graph.

iii. Compare the area of the final image with the area of $\triangle KLM$, and say whether it is the same way round or a mirror image.



Answer:

b. A phone saves a photo with the instruction "turn it 90° clockwise, then flip it left to right". A photo editor then turns the shown photo a further 180°. Put the photo's center at O, x to the right and y up. **[4]**

i. Write the matrices for the three moves.
ii. Find the single matrix for all three.
iii. Name the single move, and say whether the photo now shown is a mirror image of the scene.

c. An open question. Find two different 2×2 matrices that each map $A(2, 1)$ to $A'(1, 2)$. Is there a matrix that maps A to A' **and** $B(1, 0)$ to $B'(0, 1)$? If so, how many such matrices are there? Explain. **[3]**

4 Think **[2]**

Think, then answer.

A kaleidoscope has two mirrors at 36° to each other. Through what angle does each pair of reflections turn the picture, and how many copies of the picture fill the circle? Why do kaleidoscope makers use angles such as 45°, 36° and 30°, and not, say, 50°?